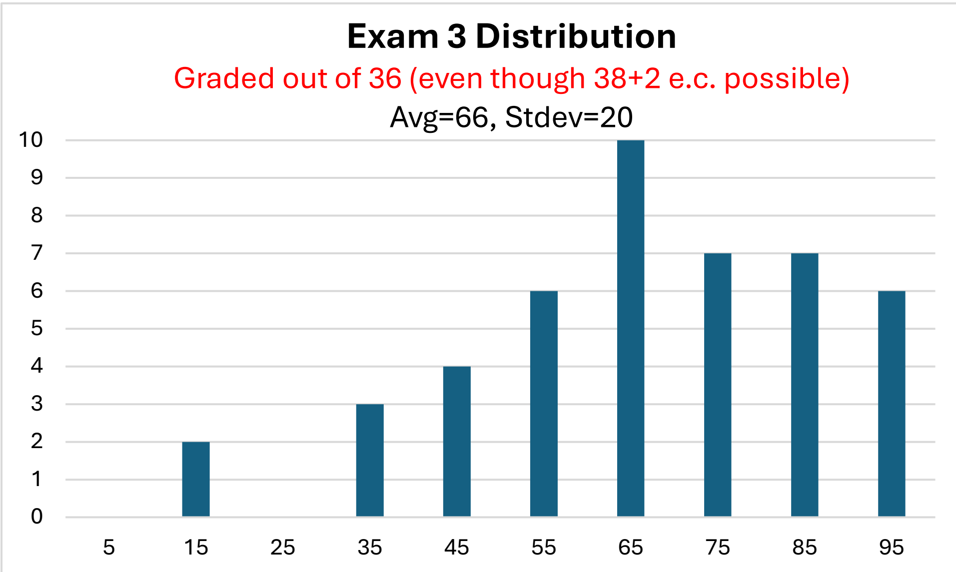
Version B, Yellow Test

162sp26t3bSoln
Distribution on this page.
Solutions begin on the next page.



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For problem 1 I did parts d & e first to help me answer parts a, b, & c.

1a) The final image was **virtual** (notice $d_i < 0$ in the ray diagram).

1b) The final image was **upright** (notice $h_i > 0$ in the ray diagram…image on same side of optic axis as object).

1c) The final image was **diminished** (notice $h_i < h_o$ in the ray diagram).

1d) A convex mirror has a NEGATIVE radius of curvature.

$$f = \frac{R}{2} = -25.0 \text{ cm}$$

Now do standard $\frac{1}{f}$ equation math.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{d_o - f}{f d_o}$$

$$d_i = \frac{f d_o}{d_o - f}$$

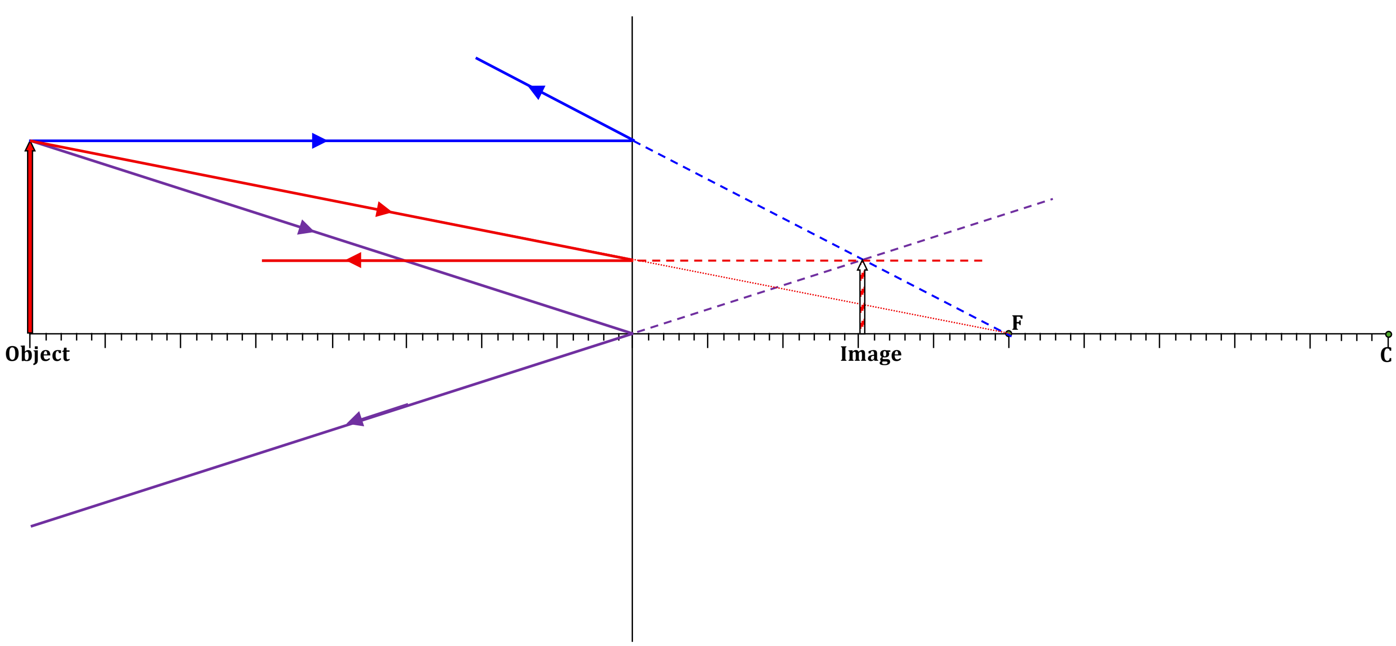
$$d_i = -15.38 \text{ cm}$$

$$M = -\frac{d_i}{d_o} = -\frac{f}{d_o - f} = \mathbf{0.385}$$

1e) Blue ray goes in parallel and goes out through focal point. Back extend the *reflected* ray.

Red ray goes in directed towards focal point and goes out parallel. Back extend the *reflected* ray.

Purple ray impacts center of the mirror and reflects using $\theta_{incidence} = \theta_{reflected}$. Back extend the *reflected* ray.



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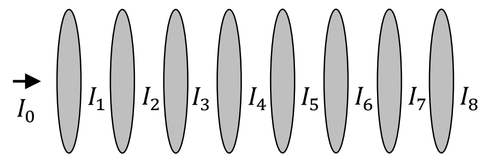


2a) Unpolarized light impacts the first polarizer (oriented vertically).

$$I_1 = \frac{1}{2} I_0$$

The next polarizer is rotated θ from the vertical.

$$I_2 = I_1 \cos^2\theta = \frac{1}{2} I_0 \cos^2\theta$$

The next polarizer is rotated and 2θ from the vertical.
Notice the angle between polarizers 1 & 2 is θ…not 2θ!

$$I_3 = I_2 \cos^2\theta = \frac{1}{2} I_0 \cos^4\theta$$

The pattern continues until we find

$$I_8 = \frac{1}{2} I_0 \cos^{14}\theta$$

Notice we do NOT get $\cos^{16}\theta$! The first polarizer chopped intensity in half!

$$2\frac{I_8}{I_0} = \cos^{14}\theta$$

$$\cos\theta = \left(2\frac{I_8}{I_0}\right)^{1/14}$$

$$\theta = \cos^{-1}\left[\left(2\frac{I_8}{I_0}\right)^{1/14}\right]$$

Final transmitted intensity is $I_8 = 33.3\%\ of\ I_0 = 0.333 I_0$!

$$\theta = \cos^{-1}\left[(2 \times 0.333)^{1/14}\right]$$

$$\theta = \cos^{-1}\left(0.666^{1/14}\right)$$

$$\theta = \cos^{-1}(\pm 0.9714)$$

$$\theta = \pm 13.74° \quad \text{or} \quad \pm 166.26°$$

Typically we specify angles between 0° & 180° for polarizers.
As such it might be better to write this answer as

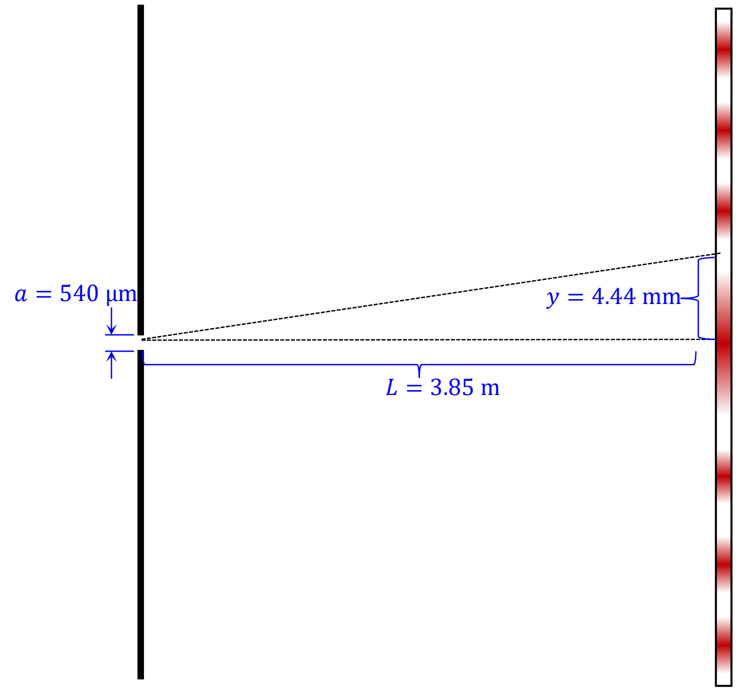
$$\boldsymbol{\theta = 13.74° \quad \text{or} \quad 166.3°}$$

This *shouldn't* be one of those problems where you check the angle $90° - \theta = 76.3°$…but I'll check it anyways for completeness.

$$\cos(76.3°) = 0.237 \neq \cos(13.74°) = 0.971$$

We got all the angles…

3a) Draw a sketch.



$$a \sin\theta_{dark} = \pm m\lambda$$

At the first min, use $m = +1$.
Minus sign works if $\theta_{dark} < 0$.

Check: is the angle small or not?

$$\tan^{-1}\left(\frac{4.44 \times 10^{-3}\ \text{m}}{3.85\ \text{m}}\right) \approx 0.07\,° \ll 10°$$

For small angles

$$\sin\theta_{dark} \approx \frac{y}{L}$$

Put it all together.

$$\frac{ay}{L} = \lambda$$

$$\lambda = \frac{(540 \times 10^{-6}\ \text{m})(4.44 \times 10^{-3}\ \text{m})}{3.85\ \text{m}}$$

$$\boldsymbol{\lambda = 62\underline{2}.8\ \text{nm}}$$

3b) Think: for part b we are using $y' = 1.234$ mm.
I'm expecting intensity is probably above 50% since we are closer to the central max than the first min.

$$I = I_0\left(\frac{\sin\beta}{\beta}\right)^2 \quad \text{where} \quad \beta = \frac{\pi a}{\lambda}\sin\theta \approx \frac{\pi a}{\lambda}\cdot\frac{y'}{L}$$

WATCH OUT! Just because the small angle approximation applies for θ, do NOT assume that small angle approximation applies for β.
WATCH OUT! Recall this equation only makes sense if we use β in units of radians.

$$\beta \approx \frac{\pi(540 \times 10^{-6}\ \text{m})}{62\underline{2}.8 \times 10^{-9}\ \text{m}}\cdot\frac{1.234 \times 10^{-3}\ \text{m}}{3.85\ \text{m}} = 0.8731\ \text{rad}$$

Because we want to express the final answer as a percent, divide both sides by I_0 now to save effort.

$$\frac{I}{I_0} = \left(\frac{\sin\beta}{\beta}\right)^2$$

$$\frac{I}{I_0} = \left[\frac{\sin(0.8731\ \text{rad})}{0.8731\ \text{rad}}\right]^2$$

$$\frac{I}{I_0} = \left[\frac{0.7663}{0.8731}\right]^2$$

$$\frac{I}{I_0} = 0.7704$$

$$\boldsymbol{\frac{I}{I_0} = 77.0\%}$$

3c) The double slit pattern would have its first min at $d\sin\theta_{dark} = \frac{\lambda}{2}$. Setting $d = 2a$ gives $a\sin\theta_{dark} = \frac{\lambda}{4}$.
This implies the new central max width is $\frac{1}{4}$ the width of the single slit pattern central max.

3d) Point **A** was located at the first min of the single slit pattern. Even if the double slit pattern causes a max at point **A**, the single slit envelope function will extinguish it! Point **A** remains dark.

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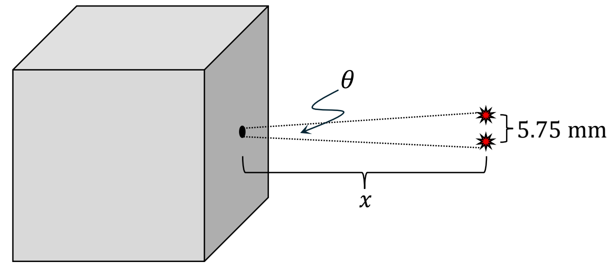
4) The problem discusses the resolution of two sources by a circular aperture.
Note: a more detailed version of the subtle points of this derivation was provided in the solution to workbook problem **36.14**.

$$\frac{1.22\lambda}{d} = \theta_{min}$$

$$\frac{1.22(650 \times 10^{-9}\text{ m})}{2.22 \times 10^{-3}\text{ m}} = \frac{5.75 \times 10^{-3}\text{ m}}{x}$$

$$x = 16.10\text{ m}$$



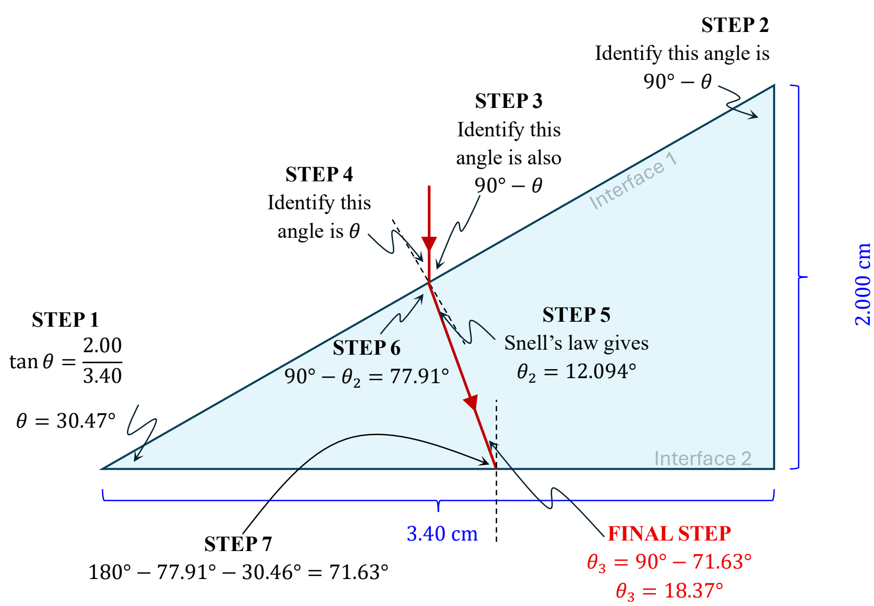
5a) When light enters a medium of higher index the beam deflects *towards* the normal.
The best answer is "**deflection right**" (see figure below).

5b) There are many ways to identify the initial angle is $\theta = 30.46°$. The method shown below isn't the fastest way to get there but it seemed the least cluttered way to label things. Hope this solution makes sense.
In step 5 I used

$$n_{air} \sin\theta = n_{diamond} \sin\theta_2$$

$$\theta_2 = \sin^{-1}\left[\frac{(1.000)\sin(30.46)}{2.42}\right] = 12.094°$$

I didn't indicate rounding digits along the way but my final answer does meet our sig fig conventions for the test.



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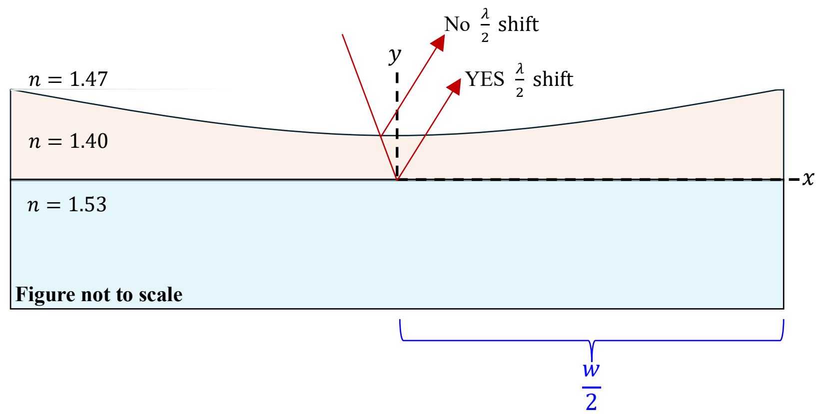
6a) We desire no reflection at the center.
We get one phase reversal from reflections.
The constructive interference condition is

$$2n_{film}t = \left(m + \frac{1}{2}\right)\lambda$$

The *minimum* thickness uses $m = 0$.

$$2n_{film}t = \frac{\lambda}{2}$$



Notice $x = 0$ at the center.
Notice film thickness at the center is thus

$$t = y(0) = a + b(0)^2 = a$$

Plug in $t = a$ and solve for a.

$$2n_{film}a = \frac{\lambda}{2}$$

$$a = \frac{\lambda}{4n_{film}}$$

$$a = 142.86 \text{ nm} \approx \mathbf{142.9\ nm}$$

6b) We were asked to make the edge of the film 7.00 times as thick as the center (figure not to scale).
Notice the edge of the film is at horizontal position $\frac{w}{2}$.

$$y_{edge} = a + b\left(\frac{w}{2}\right)^2$$

$$b = \frac{y_{edge} - a}{\left(\frac{w}{2}\right)^2}$$

$$b = 4\left(\frac{y_{edge} - a}{w^2}\right)$$

$$b = 4\left(\frac{7.00a - a}{w^2}\right)$$

$$b = \frac{24.0a}{w^2}$$

$$b = \frac{24.0(142.86 \times 10^{-9} \text{ m})}{(0.0888 \text{ m})^2}$$

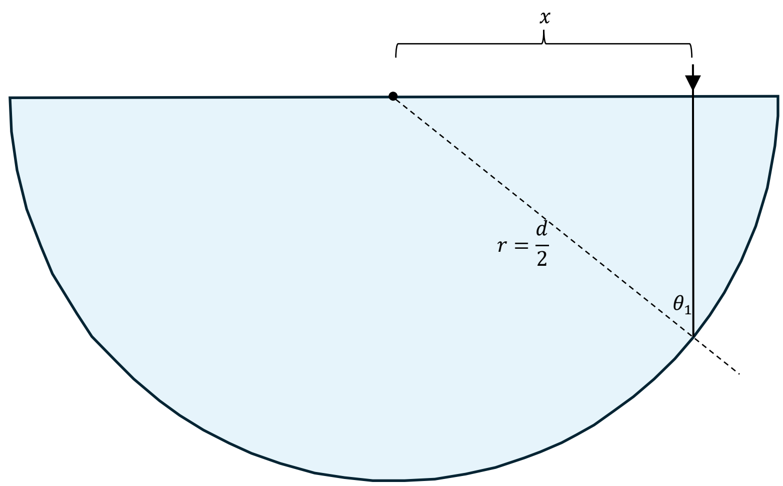
$$\boldsymbol{b = 4.35 \times 10^{-4}\ \frac{1}{m}}$$

6c) At the edge, film thickness is $7a = 7\left(\frac{\lambda}{4n_{film}}\right)$. For *this* problem, the bright & *dark* conditions are

Bright Condition: $2n_{film}t = (odd\ \#)\frac{\lambda}{2} \rightarrow t = \frac{(odd\ \#)\lambda}{4n_{film}}$	**Dark Condition:** $2n_{film}t = (even\ \#)\frac{\lambda}{2} \rightarrow t = \frac{(even\ \#)\lambda}{4n_{film}}$

Notice we expect the edge of the film should be BRIGHT (full constructive interference).

7a) More fun geometry!
Notice $\sin\theta_1 = \frac{x}{r} = \frac{2x}{d}$.



For TIR to occur

$$n_{lens}\,\sin\theta_1 = n_{air}\sin 90°$$

$$n_{lens} = \frac{1}{\sin\theta_1}$$

$$n_{lens} = \frac{d}{2x}$$

$$n_{lens} = \frac{10.00\text{ cm}}{2(3.81\text{ cm})}$$

$$\boldsymbol{n_{lens} = 1.312}$$

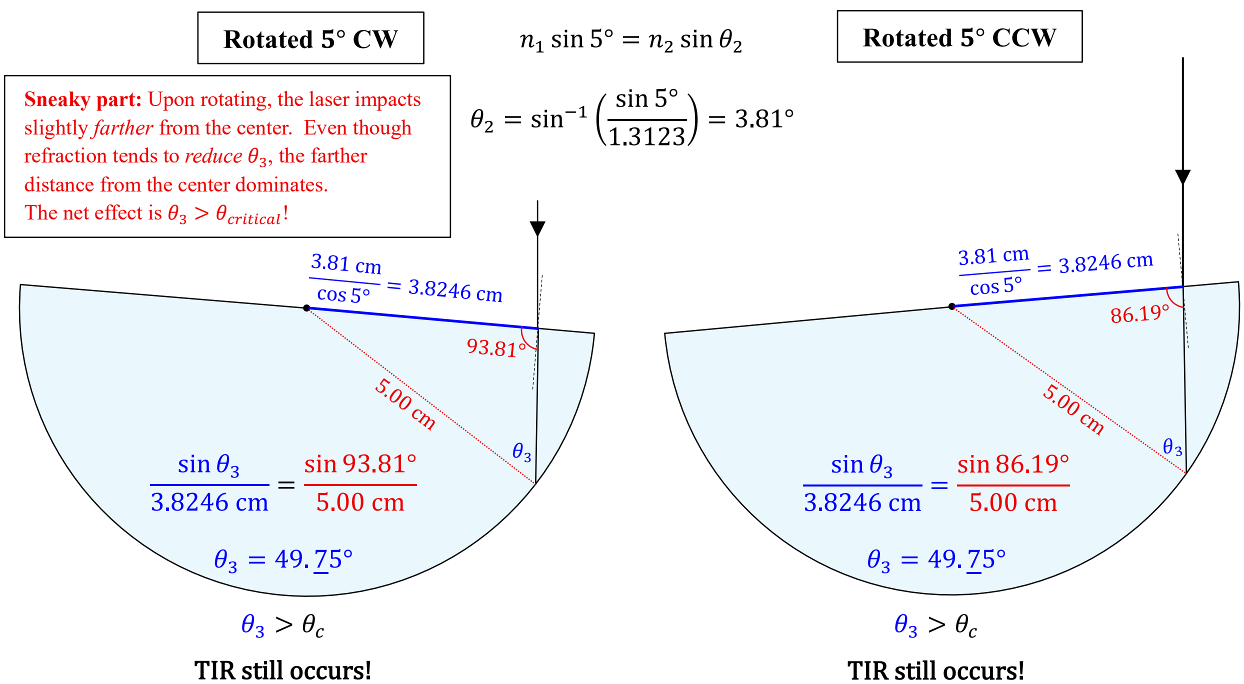
7b) **MOVE THE LASER TO THE LEFT.**

To think this through, let's first imagine what would happen if we moved the laser *to the right*.
Notice $\theta_1' > \theta_1$ if we move the laser to the right.
We know TIR occurs for angles greater than the critical angle.
Moving the laser to the right will still cause TIR.

Now imagine moving the laser *to the left*.
Notice θ_1 will get smaller.
This the same thing as saying the laser light gets *closer* to normal incidence at the curved interface.
Moving the laser to the left allows some portion of the light to be transmitted!

EC1) The best answer is "NONE OF THE OTHER ANSWERS IS CORRECT".
The new central max should be *four* times brighter!
This one is sneaky as all get out but was mentioned in a footnote on page 40 of the workbook.
The obvious guess is to assume with two slits of equal size you get twice as much light coming and thus twice as much brightness.
HOWEVER, the intensity of the light depends on the square of the electric field amplitude.
The electric field amplitude at the center of the central max doubles the light wave doubles.
This implies intensity is the *square* of the doubled field (*four* times more intense).

EC2) When I first thought about this, I wasn't sure if the effect of Snell's law might offset the effect of the rotation.
I first found the critical angle in degrees for problem 7 to be $49.\underline{6}4°$.
When I did the math, notice a really crazy thing happens: **TIR still occurs for *either* rotation direction!**



Fun side note: if you first rotate the lens then readjust the laser position such that it impacts the lens distance 3.81 cm from the center, both rotation directions allow light *to escape*.