AFTER I GIVE THE SIGNAL TO BEGIN YOU CAN REMOVE THIS SHEET. DO NOT TURN IT IN!

162sp26t4a – Once the exam has officially started, remove the top sheet. The remaining sheets comprise your exam. It is each student's individual responsibility to ensure the instructor has received her or his completed exam. Any exams not received by the instructor earn zero points. Smart watches, phones, or other devices (except scientific calculators) are not permitted during the exam.

$1\ \text{kg}\cdot\frac{\text{m}}{\text{s}^2} = 1\ \text{N}$	$1\ \text{N}\cdot\text{m} = 1\ \text{J}$	$1\ \text{Pa} = 1\frac{\text{N}}{\text{m}^2}$	$1\ \text{W} = 1\frac{\text{J}}{\text{s}}$
$c = 2.998\times 10^{8}\frac{\text{m}}{\text{s}}$	$h = 6.626\times 10^{-34}\ \text{J}\cdot\text{s}$	$h = 4.136\times 10^{-15}\ \text{eV}\cdot\text{s}$	$1.602\times 10^{-19}\ \text{J} = 1\ \text{eV}$
$hc = 124\underline{0}\ \text{eV}\cdot\text{nm}$	$\hbar = \frac{h}{2\pi} = 1.0546\times 10^{-34}\ \text{J}\cdot\text{s}$	$\hbar = \frac{h}{2\pi} = 6.583\times 10^{-16}\ \text{eV}\cdot\text{s}$	$e = 1.602\times 10^{-19}\ \text{C}$
$\sigma = 5.67\times 10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}$	$R_H = 1.09737\times 10^{7}\ \text{m}^{-1}$	$a_0 = 5.29\times 10^{-11}\ \text{m}$	$q_{proton} = +e$ & $q_{electron} = -e$
$m_e = 9.11\times 10^{-31}\ \text{kg} = 0.511\frac{\text{MeV}}{c^2}$		$m_p = 1.673\times 10^{-27}\ \text{kg} = 938\frac{\text{MeV}}{c^2}$ for this class assume $m_n = m_p$	
$\omega = \frac{2\pi}{T} = 2\pi f$	$k = \frac{2\pi}{\lambda}$	$E_\gamma = hf = \frac{hc}{\lambda}$ $\quad c = f\lambda$	$c = nv \quad \lambda_n = \frac{\lambda}{n} \quad f_n = f$
$p = \frac{E_\gamma}{c} = \frac{h}{\lambda} = \hbar k$	$I_{avg} = \frac{\mathcal{P}}{A}$	$\beta = \frac{v}{c}$	$\gamma = \frac{1}{\sqrt{1-\beta^2}}$
$\Delta t = \gamma\Delta t_0$	$L = \frac{L_0}{\gamma}$	$slope = \frac{1}{\beta}$	$t' = \gamma\left(t - \frac{vx}{c^2}\right)$
$x' = \gamma(x - vt)$	$u_x = \frac{u_x' + v}{1 + vu_x'/c^2}$	$u_y = \frac{u_y'/\gamma}{1 + vu_x'/c^2}$	$\Delta s^2 = (\Delta x^2 + \Delta y^2 + \Delta z^2) - c^2\Delta t^2$
$\vec{p} = \gamma m_{rest}\vec{u}$	$K = (\gamma - 1)E_{rest}$	$E_{rest} = m_{rest}c^2$	$E_{total} = K + E_{rest} = \gamma m_{rest}c^2$
$\mathcal{P}_{emitted} = \sigma A e T^4$	$f_{observed} = f_{source}\sqrt{\frac{1-\beta}{1+\beta}}$ Source & observer separate $\Rightarrow \beta > 0$.		$E_{total}^2 = p^2c^2 + (m_{rest}c^2)^2$
$\lambda_{max}T = 2.898\times 10^{-3}\ \text{m}\cdot\text{K}$	$E_n = nhf$	$I(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left[\exp\left(\frac{hc}{\lambda k_B T}\right) - 1\right]}$	$(1\pm\delta)^n = 1 \pm n\delta + \frac{n(n-1)}{2}\delta^2$
$\phi_{zinc} = 4.31\ \text{eV}$	$\phi_{copper} = 4.70\ \text{eV}$	$\phi_{sodium} = 2.46\ \text{eV}$ $\quad \phi_{aluminum} = 4.08\ \text{eV}$	$\phi_{iron} = 4.50\ \text{eV}$
$K_{max} = hf - \phi$	$K_{max} = e\Delta V_s$	$hf_c = \phi$	$\lambda' - \lambda_0 = \lambda_C(1 - \cos\theta)$
$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$	$E_n = -\frac{13.606\ \text{eV}}{n^2}$	$r_n = n^2 a_0$	$\lambda_C = \frac{h}{m_e c} = 2.426\times 10^{-12}\ \text{m}$
$mvr = n\hbar$	$F = \frac{k_e\lvert q_1\rvert\lvert q_2\rvert}{r^2}$	$U = \frac{k_e q_1 q_2}{r}$	$k_e = 8.99\times 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$
$v_{phase} = v_{crest} = \frac{\omega}{k}$	$p = \frac{h}{\lambda} = \hbar k$	$\Delta x\Delta p_x \geq \frac{\hbar}{2}$	$\Delta E\Delta t \geq \frac{\hbar}{2}$
$v_{group} = v_{packet} = \frac{d\omega}{dk}$	$\text{Probability} = \int_{x_1}^{x_2} P(x)\,dx$	$1 = \int_{-\infty}^{+\infty} P(x)\,dx$	$\langle f(x)\rangle = \int_{-\infty}^{+\infty} \psi^*(x)f(x)\psi(x)\,dx$
$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi n x}{L}\right)$	$E_n = \left(\frac{h^2}{8mL^2}\right)n^2$ for $n = 1, 2, \ldots$	$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + U\right)\psi = E\psi$	$0 = \int_{-\infty}^{+\infty}\psi_m^*(x)\psi_n(x)\,dx$ if $m \neq n$
$\hat{p} = i\hbar\frac{d}{dx}$	$\hat{K} = \frac{\hat{p}^2}{2m}$	$\sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$	$\sigma_p = \sqrt{\langle\hat{p}^2\rangle - \langle\hat{p}\rangle^2}$
$\sigma_x\sigma_p \geq \hbar/2$	$\psi(x) = Ae^{ikx}$	$U(x) = \frac{1}{2}m\omega^2x^2$	$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ for $n = \mathbf{0}, 1, 2, \ldots$
$c = \frac{m\omega}{\hbar}$	$\psi_0 = \left(\frac{c}{\pi}\right)^{1/4} e^{-cx^2/2}$	$\psi_1 = \left(\frac{c}{\pi}\right)^{1/4}\sqrt{2c}\,xe^{-cx^2/2}$	$\psi_2 = \left(\frac{c}{\pi}\right)^{1/4}\frac{1}{\sqrt{2}}(2cx^2 - 1)e^{-cx^2/2}$
		$T \approx e^{-2\alpha L}$ where $E = \frac{\hbar^2k^2}{2m}$ & $\alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}}$	
$n = 1,2,3,\ldots$	$\ell = 0,1,2,\ldots,(n-1)$	$m_\ell = -\ell, -\ell+1, \ldots, 0, \ldots, \ell-1, \ell$	$m_s = \pm 1/2$
$L = \hbar\sqrt{\ell(\ell+1)}$	$L_z = m_\ell\hbar$	$S = \frac{\sqrt{3}}{2}\hbar$	$S_z = m_s\hbar$
$Radial\ p.d.f. = (\psi^*\psi)4\pi r^2$	$\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$	$\psi_{200} = \frac{1}{4\sqrt{2\pi a_0^3}}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$	$\psi_{210} = \frac{1}{4\sqrt{\pi a_0^3}}\cdot\frac{r}{a_0}e^{-r/2a_0}\cos\theta$

$\text{P} = 10^{15}$	$\text{T} = 10^{12}$	$\text{G} = 10^{9}$	$\text{M} = 10^{6}$	$\text{k} = 10^{3}$	$\text{c} = 10^{-2}$	$\text{m} = 10^{-3}$	$\mu = 10^{-6}$	$\text{n} = 10^{-9}$	$\text{p} = 10^{-12}$	$\text{f} = 10^{-15}$	$\text{a} = 10^{-18}$

<table>
<tr><td rowspan="7">$\psi_{1\,0\,0} = \dfrac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$</td><td rowspan="2">$\psi_{2\,0\,0} = \dfrac{1}{4\sqrt{2\pi a_0^3}} \left(2 - \dfrac{r}{a_0}\right) e^{-r/2a_0}$</td><td>$\psi_{3\,0\,0} = \dfrac{1}{81\sqrt{3\pi a_0^3}} \left[27 - 18\dfrac{r}{a_0} + 2\dfrac{r^2}{a_0^2}\right] e^{-r/3a_0}$</td></tr>
<tr><td>$\psi_{3\,1\,0} = \dfrac{\sqrt{2}}{81\sqrt{\pi a_0^3}} \left[6\dfrac{r}{a_0} - \dfrac{r^2}{a_0^2}\right] e^{-r/3a_0} \cos\theta$</td></tr>
<tr><td rowspan="2">$\psi_{2\,1\,0} = \dfrac{1}{4\sqrt{\pi a_0^3}} \cdot \dfrac{r}{a_0} e^{-r/2a_0} \cos\theta$</td><td>$\psi_{3\,1\,\pm1} = \dfrac{1}{81\sqrt{\pi a_0^3}} \left[6\dfrac{r}{a_0} - \dfrac{r^2}{a_0^2}\right] e^{-r/3a_0} \sin\theta\, e^{\pm i\phi}$</td></tr>
<tr><td>$\psi_{3\,2\,0} = \dfrac{1}{81\sqrt{6\pi a_0^3}} \cdot \dfrac{r^2}{a_0^2} e^{-r/3a_0} [3\cos^2\theta - 1]$</td></tr>
<tr><td rowspan="2">$\psi_{2\,1\,\pm1} = \dfrac{1}{8\sqrt{\pi a_0^3}} \cdot \dfrac{r}{a_0} e^{-r/2a_0} \sin\theta\, e^{\pm i\phi}$</td><td>$\psi_{3\,2\,\pm1} = \dfrac{1}{81\sqrt{\pi a_0^3}} \cdot \dfrac{r^2}{a_0^2} e^{-r/3a_0} \sin\theta\cos\theta\, e^{\pm i\phi}$</td></tr>
<tr><td>$\psi_{3\,2\,\pm2} = \dfrac{1}{162\sqrt{\pi a_0^3}} \cdot \dfrac{r^2}{a_0^2} e^{-r/3a_0} \sin^2\theta\, e^{\pm i2\phi}$</td></tr>
</table>

$\int \sin^2 kx\ dx = \frac{x}{2} - \frac{\sin(2kx)}{4k} + C$	$\int \cos^2 kx\ dx = \frac{x}{2} + \frac{\sin(2kx)}{4k} + C$
$\int x\sin^2 kx\ dx = \frac{x^2}{4} - \frac{x\sin(2kx)}{4k} - \frac{\cos(2kx)}{8k^2} + C$	$\int x\cos^2 kx\ dx = \frac{x^2}{4} + \frac{x\sin(2kx)}{4k} + \frac{\cos(2kx)}{8k^2} + C$
$\int x^2\sin^2 kx\ dx = \frac{x^3}{6} - \frac{x^2\sin(2kx)}{4k} - \frac{x\cos(2kx)}{4k^2} + \frac{\sin(2kx)}{8k^3} + C$	$\int x^2\cos^2 kx\ dx = \frac{x^3}{6} + \frac{x^2\sin(2kx)}{4k} + \frac{x\cos(2kx)}{4k^2} - \frac{\sin(2kx)}{8k^3} + C$

The following integral table assumes a is real (not complex).	
$\int_{-\infty}^{+\infty} e^{-ax^2} = \sqrt{\frac{\pi}{a}}$	$\int e^{-ax}\, dx = -\frac{e^{-ax}}{a} + C$
$\int_{-\infty}^{+\infty} x^2\, e^{-ax^2} = \frac{1}{2} \cdot \sqrt{\frac{\pi}{a^3}}$	$\int xe^{-ax}\, dx = -\frac{e^{-ax}}{a^2}(ax + 1) + C$
$\int_{-\infty}^{+\infty} x^4\, e^{-ax^2} = \frac{3}{4} \cdot \sqrt{\frac{\pi}{a^5}}$	$\int x^2 e^{-ax}\, dx = -\frac{e^{-ax}}{a^3}(a^2x^2 + 2ax + 2) + C$
$\int_{-\infty}^{+\infty} x^6\, e^{-ax^2} = \frac{15}{8} \cdot \sqrt{\frac{\pi}{a^7}}$	$\int x^3 e^{-ax}\, dx = -\frac{e^{-ax}}{a^4}(a^3x^3 + 3a^2x^2 + 6ax + 6) + C$
$\int_{-\infty}^{+\infty} x^n\, e^{-ax^2} = 0$ if $n =$ odd integer	$\int x^4 e^{-ax}\, dx = -\frac{e^{-ax}}{a^5}(a^4x^4 + 4a^3x^3 + 12a^2x^2 + 24ax + 24) + C$

Name:____

In a rest frame, an electron moves with Lorentz factor $\gamma = 3.00$.

**1a) Determine β.

1b) Determine kinetic energy of the electron. **Use units of MeV.**

1c) Determine momentum (magnitude) of the electron. **Use units of $\frac{\text{MeV}}{c}$.**

1a	
1b	
1c	

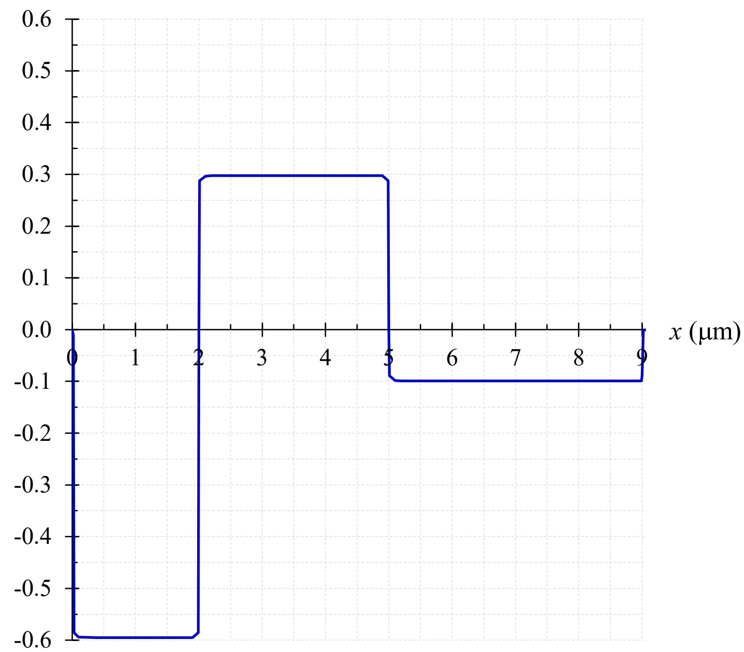
A hydrogen atom initially in the $n = 11$ state emits a photon.

***2) Determine the *shortest* possible wavelength the emitted photon could have.

2	

A normalized wavefunction for a particle in 1D is shown at right. The particle is localized between $x = 0$ and $x = 9$ μm.

3a) What axis label & units are assumed on the vertical axis of this normalized wavefunction? **Write your answer here <u>AND</u> on the plot.**



3b) Ignoring $x \leq 0$ & $x \geq 9$ nm, is there a position (or are there multiple positions) where the particle *cannot* be observed?

- **If yes,** state the positions (or ranges of positions).
- **If no,** write NONE.

3c) Estimate probability of the particle being observed between $x = 2.0$ & $x = 5.0$ μm. **Show your work for credit.

3c	

4a) Which conservation laws are used in the derivation of Compton scattering? Circle the best answer.

Only conservation of angular momentum	Only conservation of momentum	Conservation of angular momentum & energy	All 3
Only conservation of energy	Conservation of angular momentum & momentum	Conservation of momentum & energy	None of the 3

4b) Which best describes the wavelength of the scattered photon in a Compton scattering experiment?

Wavelength *always* increases	Wavelength *always* decreases	Wavelength is *always* unchanged	It depends. Sometimes wavelength increases, sometimes wavelength decreases, and, in extremely rare cases, wavelength could (in theory) remain unchanged.	There is no scattered photon!	None of the other answers is correct

Two spaceships have proper length 55.5 m and travel colinearly (along the same line). According to an observer in a rest frame, ship 1 moves to the *left* with speed $0.777c$. The same observer notices ship 2 moving to the *right* with speed $0.666c$.

5a) Which best describes the lengths of the ships observed by the rest frame observer?

$L_1 > L_2$	$L_1 = L_2$	$L_1 < L_2$	Impossible to determine without more info

5b) Which best describes the *speed* of spaceship 1 as observed by spaceship 2.

$\lvert v_{1\ rel\ 2} \rvert > c$	$\lvert v_{1\ rel\ 2} \rvert = c$	$\lvert v_{1\ rel\ 2} \rvert < c$	Impossible to determine without more info

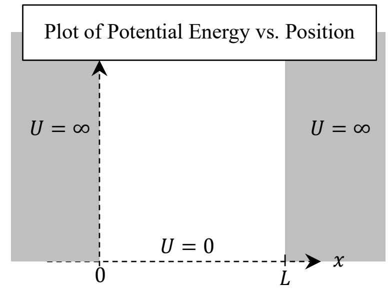
****5c) Determine the *velocity* of spaceship 1 relative to spaceship 2.

5c	

A particle in the infinite square well shown has wavefunction

$$\psi(x) = \frac{1}{\sqrt{3}}\psi_3 + \alpha\psi_4$$

where α is a positive, real constant.



****6) Determine the value of α which produces a normalized wavefunction.
If any integrals drop out, make a note indicating why they need not be calculated.

6	

**7a) Sketch <u>and label</u> an apparatus figure for the photoelectric effect. Include a photon & photoelectron. You'll know your figure is good enough if you can easily reference it while answering the questions below. Note: I don't expect your figure to look exactly like a *real* device, just a block diagram which shows what is going on.

7b) What is meant by "cut-off frequency" for a photoelectric effect experiment. Reference your figure as needed.

7c) Assuming photoelectrons are produced, how does increasing source intensity affect the *photocurrent produced*? Circle the best answer.

Has no effect	Increases current	Decreases current	Current quickly saturates to some max level	None of the other answers is correct.

7d) Assuming photoelectrons are produced, how does increasing source intensity affect the *maximum speed of photoelectrons*? Circle the best answer.

Has no effect	Increases v_{max}	Decreases v_{max}	Answer depends on work function of the metal	None of the other answers is correct.

7e) What is meant by "stopping potential" for a photoelectric effect experiment. Reference your figure as needed.

7f) What was the historical importance of the photoelectric effect? Use no more than 3-4 sentences.

8	

A calcium atom has mass 40.1 times that of a proton. The atom is confined horizontally to a 3.00 nm wide trap using laser cooling.

***8a) Determine minimum uncertainty in the atom's *horizontal velocity*.

Imagine you are studying hydrogen atom wavefunctions with a fellow student in the class. Your friend asks you to discuss an issue they discovered with $\psi_{3\,0\,0}$ given by the function

$$\psi_{3\,0\,0}(r) = \frac{1}{81\sqrt{3\pi a_0^3}}\left[27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right]e^{-r/3a_0}$$

The student says:

"I don't get it. Look at what happens when you set $r = 0$. You get $\psi(0) = \frac{1}{3\sqrt{3\pi a_0^3}}$.

If I then do $\psi^*\psi$, I get some significant non-zero probability of finding the electron at position $r = 0$…inside the nucleus.

This makes no sense because I know electrons are not found inside the nucleus!

Can you help me understand where I went wrong?"

**9) Without doing the entire calculation, show the start of the calculation & include words you'd use to explain the error to this student. If you think it helps, you can include a sketch of the actual probability distribution (optional).

A particle of mass m is subjected to a potential given by

$$U(x) = \frac{1}{2}m\omega^2 x^2$$

from $x = -\infty$ to $+\infty$. Here m & ω are positive, real constants.
The particle has the stationary state wavefunction

$$\psi(x) = Bxe^{-kx^2}$$

where B & k are positive real constants.

Do NOT normalize this wavefunction.

******10) Determine the energy value associated with this state.

- Show all work…and write neatly so I can follow it.
- During your work you are expected to first show what value of k is required.
- Hint: if done correctly, your final result for E will only include $\hbar$ & ω.
- Note: if you can't do this one, try the extra credit instead.

10	Required value for k Result for energy value

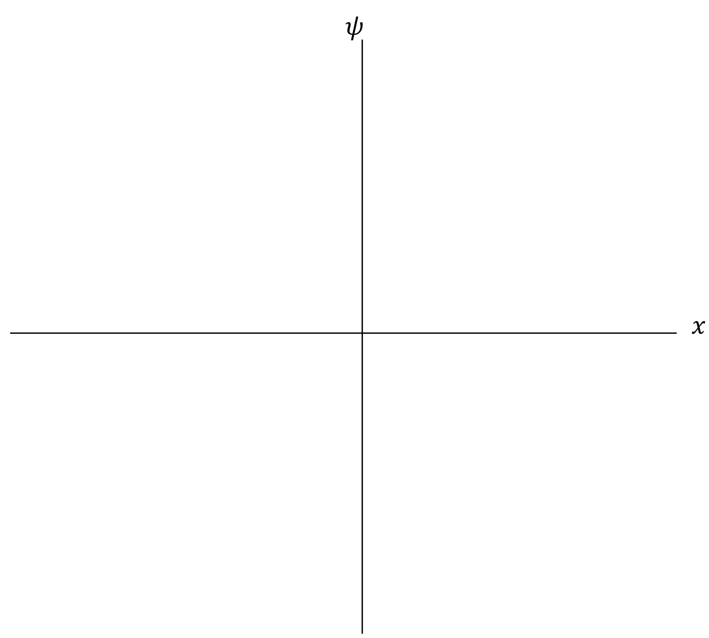
*****Extra Credit (helps you get points for problem 10 if it confused you).**

Suppose a wave function is given by the equation

$$\psi(x) = Bxe^{-kx^2}$$

This wavefunction is valid over all space (form $x = -\infty$ to $x = +\infty$). Assume k is a positive real constant. Determine the normalization constant B AND sketch a plausible plot of the wave function on the empty plot below. **Simplify your result to a decimal number with three sig figs times $\boldsymbol{k}$ raised to some power.**

$B =$



Page intentionally left blank for use as scratch paper.