

I was sitting on the beach in Baja and wanted to make a pendulum that would oscillate once every 2 seconds. Fortunately, I had a large tire, some rope, and a tree. I tied the rope around the tire and hung the tire from a branch on the tree (see figure). Assume we can model the tire as thick ring (annulus) with inner radius  $R_1=0.300$  m and outer radius  $R_2=0.400$  m. Do *not* assume the tire is a point mass. The rope has negligible mass compared to the tire. In this problem  $L$  is the distance from the branch to the inner radius of the tire. Assume the tire undergoes small angle oscillations. Figure not to scale.

\*\*\*\*1) What length of rope  $L$  is required to make the tire swing back and forth once every 2.00 seconds?

2a) Imagine you have bridge that has harmonics  $f_1, f_2, f_3$ , etc. Compared to  $f_3$ , the 2<sup>nd</sup> harmonic has a frequency that is (circle one):

Higher than  $f_3$

Lower than  $f_3$

The same as  $f_3$

Impossible to determine relationship

2b) How does the wavelength of the 5<sup>th</sup> harmonic compare to the wavelength of the 3<sup>rd</sup> harmonic?

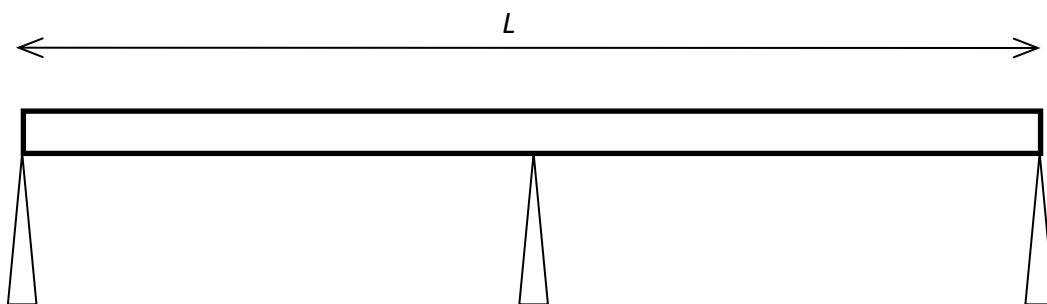
Longer than  $\lambda_3$

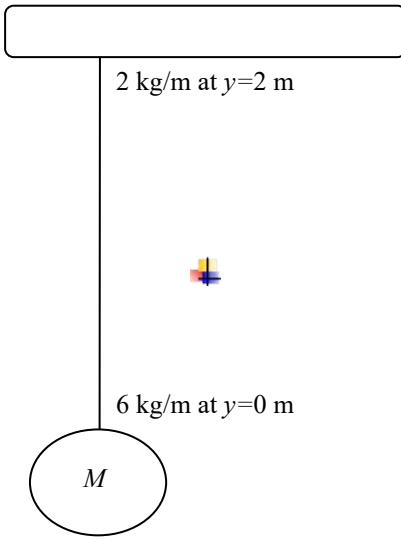
Shorter than  $\lambda_3$

The same as  $\lambda_3$

Impossible to determine relationship

\*\*\*2c) Assume the bridge is attached to the ground at three points as shown in the figure. Determine the expression describing the various harmonics  $f_n$  in terms of the speed of waves in the bridge  $v$ , a number  $n$  and the length of the bridge  $L$ . Be sure to specify what values the number  $n$  can take.

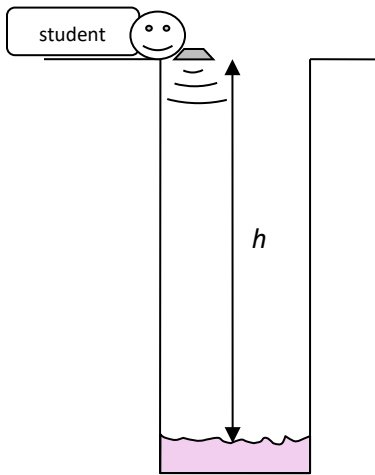




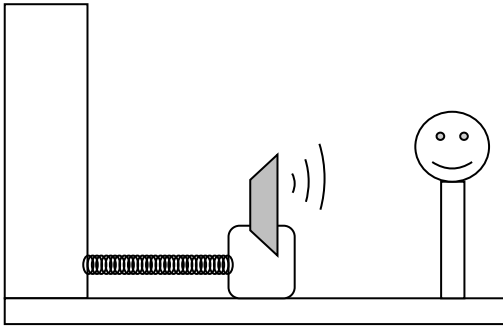
A wire is 2.00 m long and hangs vertically from the ceiling. A mass of  $M=5000$  kg is supported by the wire hanging from the ceiling. The wire has a mass per unit length of 2 kg/m at the top end and 6 kg/m at the bottom end. The mass per unit length varies linearly from the bottom to the top.

**\*\*3a)** Determine an equation that describes the mass per unit length as a function of the distance from the ceiling.

**\*\*\*\*3b)** The total mass of the wire is much less than the hanging mass  $M$ . Determine an expression for the time it would take a wave to travel from the bottom of the wire to the top of the wire. Remember that in this case the mass per unit length is not a constant!



\*\*\*\*4) A student uses an audio oscillator of adjustable frequency to measure the depth  $h$  of a water well. The student reports hearing successive resonances at 65 Hz and 75 Hz. Assuming the speed of sound is 340 m/s, how deep is the well?



A block with a speaker bolted to it is connected to a spring having spring constant  $k=27.0$  N/m. The total mass of the block and speaker is 0.0300 kg. The amplitude of the unit's motion is 0.500 m. Friction between the floor and the block is negligible. The speaker emits sound waves of frequency 440 Hz. Assume the speed of sound is 340 m/s for this problem. The minimum distance of separation between the person and the moving speaker is 1.00 m.

\*\*\*5a) Determine the highest and lowest frequencies heard by the person to the right of the speaker.

\*\*\*5b) The maximum sound level heard by the person is 60.0 dB when the speaker is closest to the observer (1.00 m away). What is the minimum sound intensity level heard by the observer?

\*\*Extra Credit: One example of a damped, driven oscillator is the old story of a bunch of soldiers marching in time as they cross a bridge (and they cause the bridge to collapse). Suppose you have a driving force with amplitude  $10^3$  N (a bunch of soldiers' footsteps) and a bridge with mass  $m=10^4$  kg. The bridge can be modeled as a stiff spring with  $k=20000$  N/m. You are told the damping constant of the bridge is 14 kg/s. The soldiers' footsteps impact the bridge at a frequency of  $f=2$  Hz.

What percent of the maximum oscillation (for the given  $m$ ,  $k$ , and  $b$ ) is caused by the soldiers' footsteps?

\*\*Extra Credit: What fraction of energy is lost by a damped harmonic oscillator after a single oscillation? Assume the system is underdamped. Answer in terms of the variables  $b$ ,  $m$ , &  $k$ .