Ancient t2 soln

1a) Rods transfer heat at same rate. Assuming steady state as specified.

$$\mathcal{P}_1 = \mathcal{P}_2$$

$$\frac{k_1 A_1 \Delta T_1}{L_1} = \frac{k_2 A_2 \Delta T_2}{L_2}$$

Same temp diff across each rod. That cancels. Plug in info about areas & conductivities ($A_2 = 3A_1$ & $k_1 = 4k_2$).

$$\frac{(4k_2)A_1}{L_1} = \frac{k_2(3A_1)}{L_2}$$

Solve for ratio requested

$$\boldsymbol{\frac{L_2}{L_1} = \frac{3}{4}}$$

1b) Entropy change is given by

$$\Delta S = \int \frac{dQ}{T}$$

During a phase change, temp is constant. Pop

$$\Delta S = \frac{1}{T}\int dQ = \frac{Q}{T} = \frac{-m_{water}L_f}{T}$$

Notice we must use units of K for temp! Latent heat is given in units of $\frac{\text{kJ}}{\text{kg}}$ so use kg for mass.

$$\Delta S = \frac{-(0.020\ \text{kg})\left(334\,\frac{\text{kJ}}{\text{kg}}\right)}{273.15\ \text{K}} = --24.5\,\frac{\text{J}}{\text{K}}$$

1c) Net entropy change of a system is always positive for real world processes.
We are assuming the heat lost by the water was gained by the metal.
We expect the sign change of entropy is positive for the metal.
We expect the positive entropy change of the metal must be larger than $24.5\,\frac{\text{J}}{\text{K}}$.

2) Gauge pressure is pressure difference between inside the measurement chamber and outside (ambient) pressure.

$$P_{absolute} = P_{tire}$$

$$P_{gauge} = P_{tire} - P_{ambient}$$

$$P_{tire} = P_{gauge} + P_{ambient} = 55 \text{ PSI}$$

For this problem I will use unprimed for initial and primed for final to reduce subscripts.

$$\frac{P'_{tire}V'}{P_{tire}V} = \frac{n'RT'}{nRT}$$

Temp changes. Volume of the tire is constant. We let air out until $P'_{tire} = P_{tire}$. At this point, $n' < n$.

$$\frac{1}{1} = \frac{n'T'}{nT}$$

Guess I didn't need to do all that work converting gauge pressure to absolute…but it is good practice.

When using $PV = nRT$ you must use absolute pressure and temperature in K.

Rearrange the equation to solve for n'.

$$n' = n\frac{T}{T'}$$

How do I get "fraction of air to let out"?

Think: this is essentially the same as getting percent change in air. Only difference: do not convert to %.

$$\frac{\Delta n}{n} = \frac{n' - n}{n}$$

$$\frac{\Delta n}{n} = \frac{\left(n\frac{T}{T'}\right) - n}{n}$$

$$\frac{\Delta n}{n} = \frac{T}{T'} - 1$$

$$\frac{\Delta n}{n} = \frac{293.15}{318.5} - 1$$

$$\frac{\Delta n}{n} = -0.0786$$

Notice I had to use units of K because the original equation ($PV = nRT$) didn't use a ΔT.

The minus sign indicates you *decrease* the number of moles by $0.0786 \approx \frac{8}{100} = 8\%$.

3) **WATCH OUT!** Chart wants work done ON the gas. In class we have been using work done BY the gas.

Recall from class & exam equation sheet:

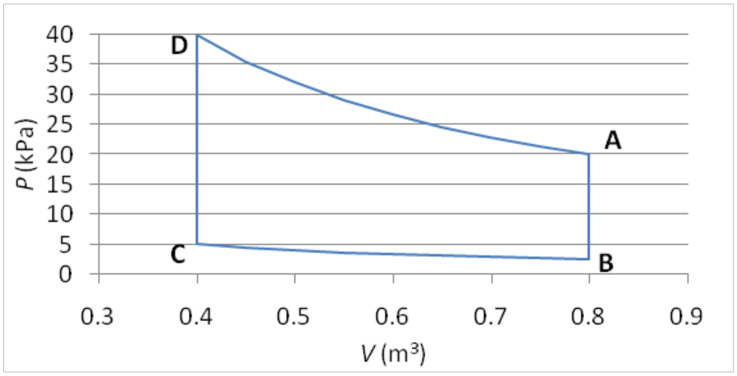
$$W_{on} = -W_{by}$$

Recall this makes the first law

$$\Delta E_{int} = Q_{in} + W_{on}$$

instead of

$$\Delta E_{int} = Q_{in} - W_{by}$$



Most of this cycle uses $f = 5$ & $\gamma = \frac{7}{5} = 1.4$.

Clockwise cycle: expect $W_{\substack{by \\ NET}} > 0$ & thus $W_{\substack{on \\ NET}} < 0$.

Checks to determine type of process for **D** to **A**:

Isothermal?	**Adiabatic?**
$P_A V_A = 16$ $P_D V_D = 16$	$P_A V_A^{\gamma} = 14.6$ $P_D V_D^{\gamma} = 11.1$
When doing these checks, you can use whatever units you want (as long as you use the same units for each point).	

Determine pressure at B precisely using ratio of ideal gas law equations with point C.
Why use point C? We know process B to C is isothermal. Temperature (unknown) will drop out!

$$\frac{P_B V_B}{P_C V_C} = \frac{n_B R T_B}{n_C R T_C} = 1 \quad \rightarrow \quad P_B = \frac{P_C V_C}{V_B} = 2.5 \text{ kPa}$$

Finally, let's think about the units of PV to save some time:

$$1 \text{ kPa} \cdot 1 \text{ m}^3 = 1000 \text{ Pa} \cdot \text{m}^3 = 1000 \text{ J} = 1 \text{ kJ}$$

Process	**Process Type**	***Work done on gas* (kJ)**	***Q added to gas* (kJ)**	**Δ*E of gas* (kJ)**
A to B	isochoric	0	$Q_{in} = \Delta E_{int} - W_{on} = -35.0$	$\frac{f}{2}(P_f V_f - P_i V_i) = -35.0$
B to C	isothermal	$-nRT \ln \frac{V_f}{V_i} = PV \ln \frac{V_i}{V_f} = +1.386$ Check: Compression ⇒ $W_{on\ gas} > 0$.	$Q_{in} = \Delta E_{int} - W_{on} = -1.386$	0
C to D	isochoric	0	$Q_{in} = \Delta E_{int} - W_{on} = +35.0$	$\frac{f}{2}(P_f V_f - P_i V_i) = +35.0$
D to A	isothermal	$-nRT \ln \frac{V_f}{V_i} = PV \ln \frac{V_i}{V_f}$ $= -11.09$ Check: Expansion ⇒ $W_{on\ gas} < 0$.	$Q_{in} = \Delta E_{int} - W_{on} = +11.09$	0
For entire cycle		-9.704	$+9.704$	0

Solution continues on the next page…

3b) For practice you should draw an engine diagram (there is one on your exam).
You are expected to know what the arrows represent on an engine diagram.
I will do that at the bottom of this page.
Try to do that yourself before looking at the solution.
I used that engine diagram to compute an efficiency of $\mathbf{21.04\%}$.

3c) The engine is only able to use 22.56% percent input energy.

$$\mathcal{P}_{output} = \frac{Work}{time} = 20.0\text{ W}$$

Since we know net work in a single cycle is 10.40 kJ, we can compute the time of a cycle using

$$\frac{Work_{1\,cycle}}{time_{1\,cycle}} = 20.0\text{ W} \quad \rightarrow \quad time_{1\,cycle} = \frac{Work_{1\,cycle}}{20.0\text{ W}} = 520\text{ s}$$

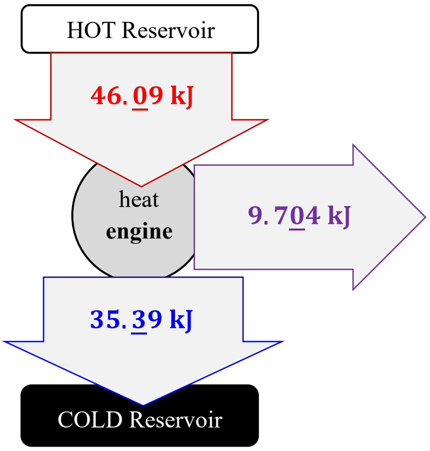
This has got to be the most ridiculous engine design ever…but it made for easy math in the chart.
This happens in a physics class sometimes. I digress…back to the problem at hand…
Energy input is in the form Q_{hot}.
Power input required per second is thus

$$\mathcal{P}_{in} = \frac{Q_{hot\ 1\ cycle}}{time_{1\,cycle}} = 88.6\text{ W}$$

Another way to see this is to use the efficiency equation in another form and divide all terms by $time_{1\,cycle}$.

$$\eta = \frac{W_{NET}}{Q_{in}} = \frac{\frac{W_{net}}{t_{cycle}}}{\frac{Q_{hot}}{t_{cycle}}} = \frac{\mathcal{P}_{output}}{\mathcal{P}_{input}} \quad \rightarrow \quad \mathcal{P}_{output} = \eta \mathcal{P}_{input} \quad \rightarrow \quad \mathcal{P}_{input} = \frac{1}{\eta}\mathcal{P}_{output} = 88.7\text{ W}$$

Notice the rounding errors in the third sig fig. Clearly I used a rounded intermediate answer. BAD ROB!
It is good to see this *before* the test. Be sure you keep an extra digit *past the rounding digit* for all computations.
This prevents confusion & wasted time on test day.



The *upper* arrow uses Q_{hot} = sum all *positive* $Q'_{in}s$ = $\mathbf{46.\underline{0}9}$ **kJ**

The *middle* arrow uses $W_{NET\ BY}$ = $\mathbf{+9.7\underline{0}4}$ **kJ**

The *lower* arrow uses $|Q_{cold}|$ = |sum all *negative* $Q'_{in}s$| = $\mathbf{36.\underline{3}9}$ **kJ**

$$\textbf{Efficiency} = \mathbf{1} - \frac{|Q_{cold}|}{Q_{Hot}} = \mathbf{21.\underline{0}4}\%$$

4a) In a physics class, unless otherwise specified, the default units for pressure and radius are Pa & m respectively. Start with the term inside the parens.
the units on any one term in a sum (or difference) must match the units of each & every other term in the sum.
Said another way: to add two terms together the two terms must have the same units.

$$[\boldsymbol{b}] = [\boldsymbol{r}] = \mathbf{m}$$

WATCH OUT! As you saw on test 1, if you wrote $b = \mathrm{m}$ I will take off points.
There is a difference between b (the number with units) and $[b]$ (the units only).

Again, units on any one term in a sum (or difference) must match the units of each & every other term in the sum.

$$[\boldsymbol{c}] = [\boldsymbol{P}] = \mathbf{Pa}$$

Now compare units of pressure to the units of the first term on the right hand side.
Again, units on any one term in a sum (or difference) must match the units of each & every other term in the sum.

$$[P] = [a] \cdot [r - b]^2$$

$$[\boldsymbol{a}] = \frac{\mathbf{Pa}}{\mathbf{m^2}} = \frac{\mathbf{N}}{\mathbf{m^4}}$$

Either of these two forms is reasonable for a physics exam.

4b) Because we are asked for the magnitude of work done, we were asked for a positive quantity.
We need not concern ourselves with signs (W_{by} versus W_{on}) as long as we ensure the final result is positive.

$$W = \left| \int_i^f [a(r+b)^2 - c] \, dV \right|$$

Think: how does volume change (dV) relate to radius change (dr)?

$$V = V_{sphere} = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$dV = 4\pi r^2 \, dr$$

As the balloon in inflated, it increases in size from radius 0 to final radius $r_f = \frac{D}{2}$.
These will be in the limits of integration!

$$W = \left| \int_0^{D/2} [a(r+b)^2 - c] \, 4\pi r^2 \, dr \right|$$

$$W = \left| 4\pi \int_0^{D/2} [a(r^2 + 2rb + b^2) - c] \, r^2 \, dr \right|$$

$$W = \left| 4\pi \int_0^{D/2} [\, ar^4 + 2abr^3 + (ab^2 - c)r^2] \, dr \right|$$

$$W = \left| 4\pi \left[\frac{ar^5}{5} + \frac{1}{2}abr^4 + \frac{1}{3}(ab^2 - c)r^3 \right]_0^{D/2} \right|$$

ALWAYS CHECK THE ZERO LIMIT JUST IN CASE…you have been warned.
Let's get real…a ton of functions (e.g. cosine or e^x do not got to zero at the zero limit). It is on you to pay attention.
That said, in this case the zero limit does drop out.

$$W = \left| 4\pi \left[\frac{aD^5}{160} + \frac{1}{32}abD^4 + \frac{1}{24}(ab^2 - c)D^3 \right] \right|$$

$$W = \left| \frac{\pi}{120} [3aD^5 + 15abD^4 + 20(ab^2 - c)D^3] \right|$$

5a) Heat absorbed relates to temperature change. We need to know temperature change to determine ΔL.
Do work algebraically in case it ends up cancelling something out…might save us headaches with units later…

$$Q = mc\Delta T$$

$$\Delta T = \frac{Q}{mc}$$

To get mass use area density. If you haven't used it in a while, it should be obvious from the units

$$m = \sigma A$$

where σ is *area* mass density (from physics 161) and A is area of the plate.

$$\Delta T = \frac{Q}{\sigma A c}$$

Length change is given by

$$\Delta L = L_0 \alpha \Delta T$$

Plug in algebraic result for ΔT from the $Q = mc\Delta T$ eqt'n.

$$\Delta L = \frac{L_0 \alpha Q}{mc}$$

$$\Delta L = \frac{(0.250\ \text{m})\left(15 \times 10^{-6}\,\frac{1}{{}^\circ\text{C}}\right)(20\ \text{J})}{\left(25.0\,\frac{\text{kg}}{\text{m}^2}\right)(0.250\ \text{m})^2\left(400\,\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)}$$

Remember, we can use units of °C & K interchangeably in conversion factors.

$$\Delta L = 1.200 \times 10^{-7}\ \text{m} = 120.0\ \text{nm}$$

Notice this is a huge percent difference when compared to the wavelength.
In this context, it seems logical to compute the percent difference using

$$\%\ \text{diff} = \frac{length\ change}{wavelength} = \frac{\Delta L}{\lambda} \times 100\% = 20.0\%$$

Note: this is an instance where it behooves us to ignore the minus sign.
When know the plate increased in size (because $\alpha > 0$ and told plate heated up).

What is the point of this problem? When designing optical instruments, it is crucial to if & how thermal expansion will affect device operation. This is true for a lot of things (bridges, electronics, fillings in your teeth, etc). Even small amounts of heat and small temperature changes can cause catastrophic failure.

6a) **WATCH OUT!** The average of *speed squared* $\left(\overline{v^2}\right)$ is not the same thing as average of speed *squared* $\left(\bar{v}^2\right)$. Pay close attention to your phrasing and the associated symbols when discussing probability problems.

$$\bar{v} = \int_0^\infty v \left(\frac{a}{\pi}\right)^{3/2} \exp(-av^2)\ 4\pi v^2\, dv$$

$$\bar{v} = 4\pi \left(\frac{a}{\pi}\right)^{3/2} \int_0^\infty \exp(-av^2)\ v^3\, dv$$

$$\bar{v} = 4\pi \left(\frac{a}{\pi}\right)^{\frac{3}{2}} \left(\frac{1}{2a^2}\right)$$

$$\bar{v} = \frac{2}{\pi^{1/2} a^{1/2}}$$

$$\bar{v} = \frac{2}{\pi^{1/2} \left(\frac{m}{2k_B T}\right)^{1/2}}$$

$$\textbf{Average speed} = \bar{v} = \sqrt{\frac{8k_B T}{\pi m}}$$

Now consider

$$\overline{v^2} = \int_i^f v^2\, f(v)\, dv$$

$$\overline{v^2} = \int_0^\infty v^2 \left(\frac{a}{\pi}\right)^{3/2} \exp(-av^2)\ 4\pi v^2\, dv$$

$$\overline{v^2} = 4\pi \left(\frac{a}{\pi}\right)^{3/2} \int_0^\infty \exp(-av^2)\ v^4\, dv$$

$$\overline{v^2} = 4\pi \left(\frac{a}{\pi}\right)^{3/2} \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$$

$$\overline{v^2} = \frac{3}{2a}$$

$$\overline{v^2} = \frac{3}{2\left(\frac{m}{2k_B T}\right)}$$

$$\textbf{Average of speed squared} = \overline{v^2} = \frac{3k_B T}{m}$$

Now it is easy to compute standard deviation:

$$\sigma = \sqrt{\overline{v^2} - \bar{v}^2} = \sqrt{\frac{3k_B T}{m} - \left(\sqrt{\frac{8k_B T}{\pi m}}\right)^2} = \sqrt{\left(3 - \frac{8}{\pi}\right)\frac{k_B T}{m}} \approx 0.673\sqrt{\frac{k_B T}{m}}$$

6b) As temperature increases, the standard deviation (width of curve) increases. See the link below for visuals: https://demonstrations.wolfram.com/TheMaxwellSpeedDistribution/