1a) We are told the lens is *convex*. This implies focal length is positive.
A *concave* lens has *negative* focal length is negative.
You are expected to understand this distinction on exams and adjust the sign of focal lengths from the problem statement accordingly.
We are also given magnification.
Use algebra to eliminate an unknown from the $\frac{1}{f}$ equation and the magnification equation.

$$M = -\frac{d_i}{d_o} \quad \rightarrow \quad d_i = -d_o M$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{f} = \frac{1}{d_o} - \frac{1}{d_o M}$$

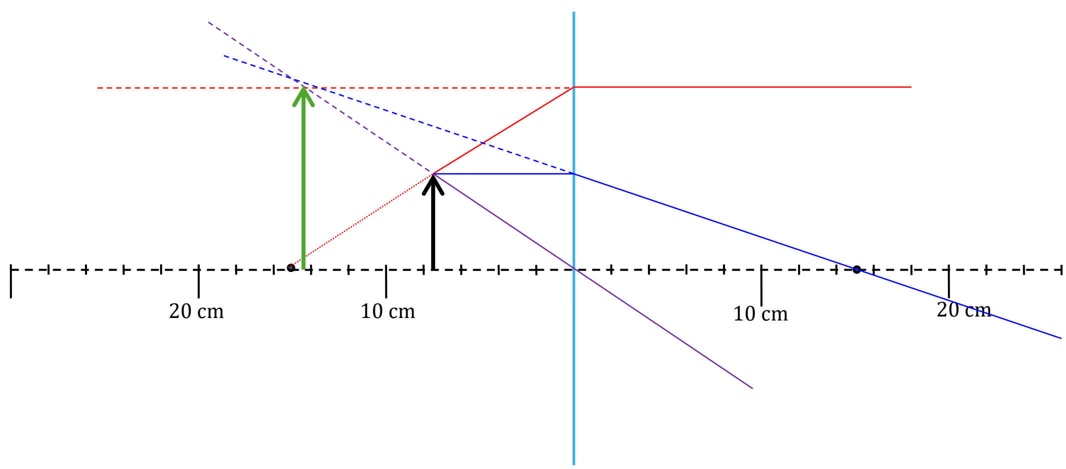
$$\frac{1}{f} = \frac{M}{M d_o} - \frac{1}{d_o M}$$

$$\frac{1}{f} = \frac{M-1}{M d_o}$$

$$d_o = f\frac{M-1}{M}$$

$$d_o = (+15\text{ cm})\frac{2-1}{2} = 7.5\text{ cm}$$

1b) Before drawing the ray diagram, determine $d_i = -15$ cm. Why do this? So I can tell where to put the lens! It also helps me check my final drawing! Notice the drawing gives a fairly decent estimate for the image and it has approximately the correct magnification. Your drawings on the test should be within about 5% percent of mine.

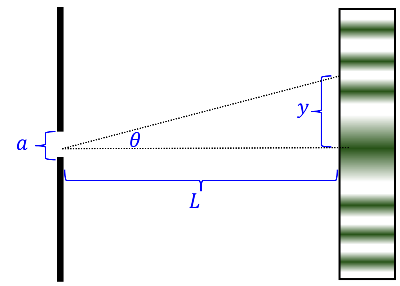


Final image is virtual, upright, and enlarged.

Other ray diagram (1bonus) is at the top of the next page.

2) The *single* slit interference pattern has *dark* fringe positions given by

$$a\sin\theta = \pm m\lambda \qquad \text{for } m = 1, 2, 3, \ldots$$



When thinking about this problem conceptually, it is usually works to assume the small angle approximation applies (unless the problem directly mentions large angles). For small angle approximation:

$$\frac{ay}{L} = \pm m\lambda$$

Solving this for fringe position y is typically most instructive.

$$y = \pm\frac{m\lambda L}{a}$$

Notice fringe spacing (and thus the scaling of the pattern) increases if wavelength is increased.
Notice the 1st order dark fringes corresponding to $m = \pm 1$ change in proportion to other fringe locations.
The central max will scale in size by the same factor as dark fringe spacing.
The best answer was e.

3) The small angle approximation is

$$\sin\theta = \tan\theta = \theta$$

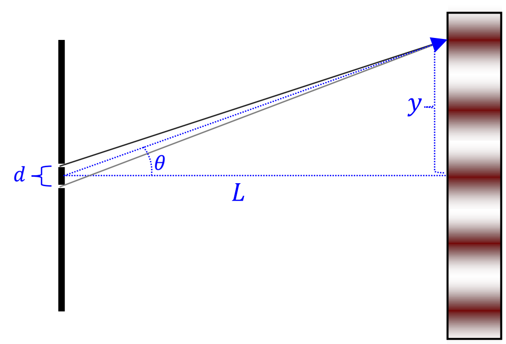
where θ must be given *in radians*. Similarly $\cos\theta \approx 1 - \frac{\theta^2}{2}$ which we can often further simplify to $\cos\theta \approx 1$.
In this class we typically say this approximately is valid for angles under 10°…sometimes 8° depending on desired precision. The precise angle required for 1% precision will vary depending on the trig function used and context. For example, the LibreTexts used max angle 11.5° for that particular application (scroll to equation 11.3.4).

4) The *double* slit pattern has *bright* fringe positions given by

$$d\sin\theta = \pm m\lambda \qquad \text{for } m = 0, 1, 2, 3, \ldots$$

In this expression, $m = 0$ corresponds to the central max.
Using the small angle approximation and solving for y gives

$$y = \pm\frac{m\lambda L}{d}$$



Again the entire pattern scales the same amount as fringe spacing.
Notice increasing d *shrinks* the pattern (causes *smaller* y).

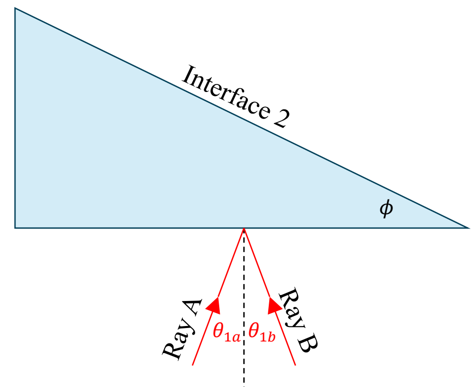
- Distance between fringes should decrease.
- The width of the central max should decrease.
- The angles from the central max to fringes to decrease.
- The overall brightness of the pattern should *increase*.
  Why? Same amount of light energy crammed into a smaller area.
  Intensity (given by power over area) is the parameter directly correlated to brightness.
  In this scenario we have the same power incident upon a smaller area…more intensity…more brightness.

5a) To experience TIR, light must be travelling in the high index material. As light hits interface two, we need an angle *larger* than the critical angle. Recall, angles in optics are measured relative to the normal to the interface. In short, this implies rays *nearly perpendicular* to an interface are *unlikely* to experience TIR while rays *nearly parallel* to an interface are *likely* to experience TIR.

For these two materials $n_{hi} = 2.00$ and $n_{lo} = 1.00$.

Critical angle is $\theta_{critical} = 30.0\ °$.

This means we want our light ray angled *more* than 30.0° to the normal of interface 2. Ray B is almost certainly going to meet this requirement while Ray A is nearly perpendicular to interface 2 (unlikely to experience TIR).



5b) Normally incident implies $\theta_{1a} = \theta_{1b} = 0°$.

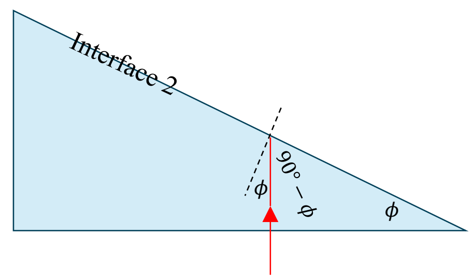
Said another way, the rays strike the bottom of the triangular prism travelling perpendicular to the bottom interface.

A ray travelling normal to an interface is transmitted with zero deflection!

A bit of geometry shows the angle of incidence at interface 2 is ϕ.

In this scenario, we require the $\phi > \theta_{critical}$.

The minimum angle to cause TIR is $\phi = 30.0°$.



5c) Consider the lower two figures at right.

We require $\theta_3 \geq \theta_{critical} = 30.0\ °$.

A bit of geometry implies

$$\theta_2 = \theta_3 - \phi$$

This implies

$$\theta_2 \geq 30.0° - 20.0° = 10.0°$$

Snell's law tells at interface 1 gives

$$n_1 \sin \theta_{1b} = n_2 \sin \theta_2$$

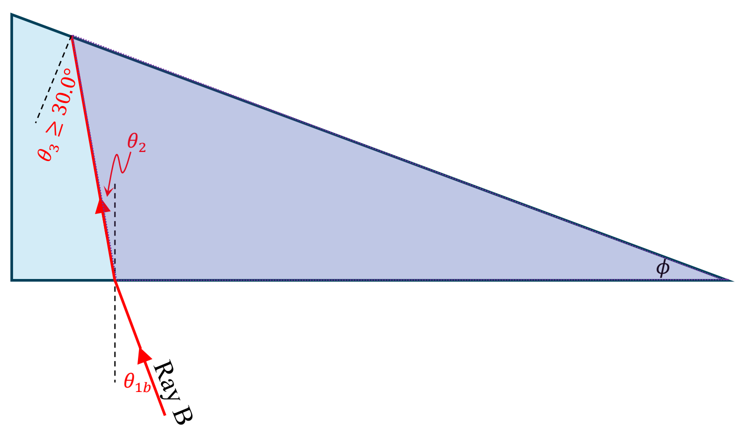
$$\boldsymbol{\theta_{1b} \geq 20.3°}$$

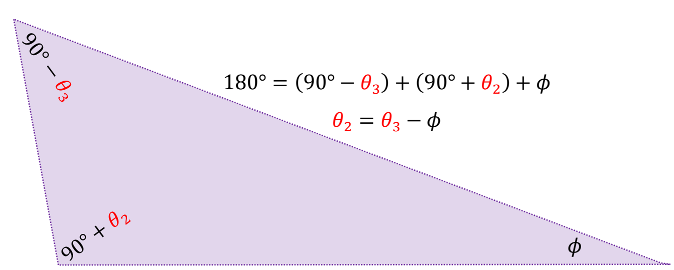
Think before blindly trusting the direction of the greater than symbol.

We require TIR at interface 2.

This implies we require larges angles for θ_3.

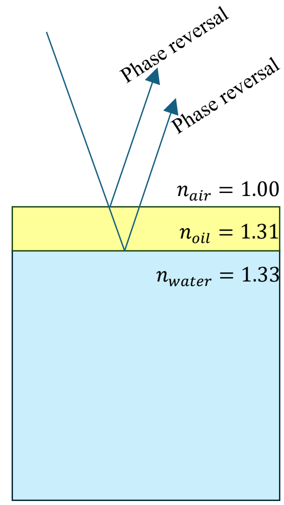
For *this* geometry problem, that implies we require large angles for θ_2 (and thus for θ_{1b} as well).





6) For a thin film problem:

- Draw a sketch
- Identify which material is the film
- Determine phase reversals & write out constructive/destructive conditions accordingly.
- If necessary, derive an expression for thickness as a function of x or r
- Re-read the question and solve for something.
- If thickness varies, interference in the zero-thickness limit is determined by the number of phase reversals



My sketch is shown at right.
This problem statement told us the film is oil (sometimes you have to think a bit more than this).
Phase reversal every time you reflect off a *higher* index of refraction.

Constructive	Destructive
$2n_{oil}t = m\lambda_{vac} \quad m = 0,1,2,\ldots$	$2n_{oil}t = \left(m + \frac{1}{2}\right)\lambda_{vac} \quad m = 0,1,2,\ldots$

This problem mentioned uniform film. Presumably that implies uniform thickness and composition.
In this case, no need to derive any function for thickness.

Problem mentions bright reflection (constructive interference) for $\lambda_1 = 450$ nm and no reflection for $\lambda_2 = 600$ nm.
While not specified, we assume wavelengths are stated as wavelength *in vacuum* (unless otherwise specified).
Since $2n_{oil}t$ is the same for both films, we know

$$m_1\lambda_1 = \left(m_2 + \frac{1}{2}\right)\lambda_2$$

$$m_1 = \left(m_2 + \frac{1}{2}\right)\frac{\lambda_2}{\lambda_1}$$

$$m_1 = \left(m_2 + \frac{1}{2}\right)\frac{600}{450}$$

$$m_1 = \left(m_2 + \frac{1}{2}\right)\frac{4}{3}$$

$$3m_1 = 4m_2 + 2$$

We were asked for minimum thickness…use the smallest set of integers that satisfy this expression.
Use trial and error: the minimum thickness must use $m_1 = 2$ and $m_2 = 1$.
Pick either m and solve for thickness in the appropriate equation (recall it was bright/constructive for m_1).

$$2n_{oil}t = m_1\lambda_{vac\,1}$$

$$t = \frac{m_1\lambda_{vac\,1}}{2n_{oil}}$$

$$t = \frac{2(450\text{ nm})}{2(1.31)}$$

$$t = 34\underline{3}.5\text{ nm}$$

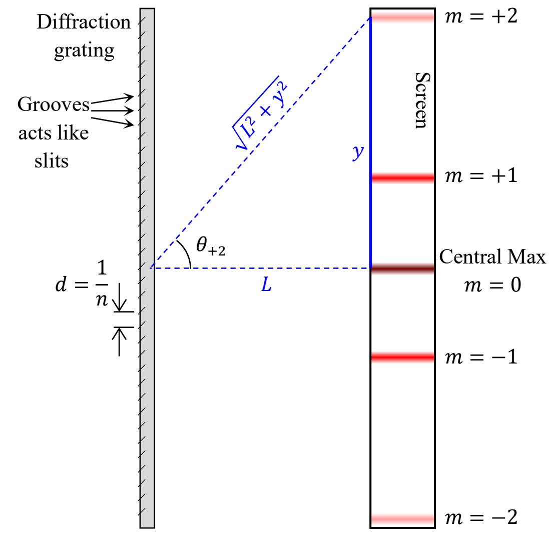
If you are wondering if this is plausible for a thin film of oil, it is.
Remember the drop of oil spreading out on water's surface into an oil slick (problem **1.30** in **Volume 1**, page 11)?
This was the demo I usually do at the very end of class 1 in PHYS 161…

https://sciencedemonstrations.fas.harvard.edu/presentations/molecular-size

Note: the Harvard website has a typo and states it is 17 nm not 1.7 nm…*everybody* makes typos!
The 0.5 µL droplet usually spreads out to approximately 60 cm giving approximate molecular thickness 2 nm.
Here is a similar demo where the droplet is measured to be about 2 nm thick: https://spark.iop.org/estimating-size-molecule-using-oil-film

7) It probably helps to make a quick sketch similar to the one at right. For a diffraction grating we know

$$d\sin\theta_{bright} = \pm m\lambda \qquad \text{where} \quad d = \frac{1}{n} \quad \text{and} \quad n = \frac{\text{\# of lines}}{\text{meter}}$$



For this problem we are using:

- $L = 60\text{ cm}$
- $y = 30\text{ cm}$
- $\lambda = 500\text{ nm}$
- $m = +2$
- $\sin\theta_2 = \frac{y}{\sqrt{L^2+y^2}}$

WATCH OUT! For a grating the small angle approximation typically does *not* apply.

$$\frac{1}{n}\sin\theta_{bright} = m\lambda$$

$$n = \frac{\sin\theta_{bright}}{m\lambda}$$

$$n = \frac{\frac{y}{\sqrt{L^2+y^2}}}{m\lambda}$$

$$n = \frac{y}{m\lambda\sqrt{L^2+y^2}}$$

$$n = \frac{0.30\text{ m}}{(2)(500\times10^{-9}\text{ m})\lambda\sqrt{(0.60\text{ m})^2+(0.30\text{ m})^2}}$$

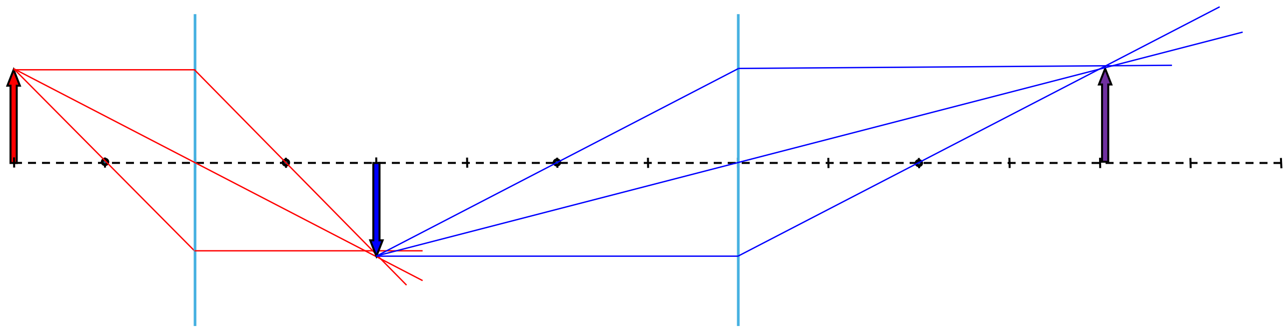
$$\boldsymbol{n = 6.67\times10^5\,\frac{\textbf{lines}}{\textbf{m}}}$$

It's common to express this result as lines per mm or lines per cm.

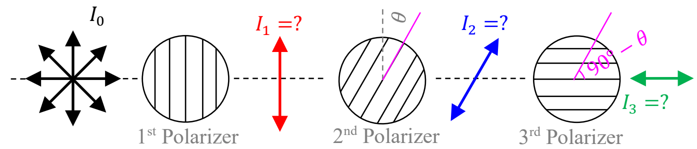
$$\boldsymbol{n = 667\,\frac{\textbf{lines}}{\textbf{mm}}}$$

On a test I would take either of these last two forms.

1bonus) This is just a fun little problem.
Notice the final image should be upright, real and be neither enlarged nor diminished ($M = 1$).
If you are doing your algebra correctly, the final image distance should be $4f$ from second lens.
Trick question!



9a) I_0 is unpolarized. Lose 50% through first polarizer.



$$I_1 = \frac{1}{2} I_0$$

Use Malus's law to determine I_2.

$$I_2 = I_1 \cos^2 \theta$$

Now add the third polarizer

$$I_3 = I_2 \cos^2(90° - \theta)$$

$$I_3 = (I_1 \cos^2 \theta) \cos^2(90° - \theta)$$

$$I_3 = \left(\left(\frac{1}{2} I_0\right) \cos^2 \theta\right) \cos^2(90° - \theta)$$

A useful trig identity for this chapter (and in statics):

$$\cos(90° - \theta) = \sin \theta$$

Using this identity gives the result

$$I_3 = \frac{1}{2} I_0 \cos^2 \theta \sin^2 \theta$$

We could simplify this even more using the identity

$$2 \sin \theta \cos \theta = \sin 2\theta \quad \rightarrow \quad \sin^2 \theta \cos^2 \theta = \frac{1}{4} \sin^2 2\theta$$

The final simplified formula is

$$I_3 = \frac{1}{8} I_0 \sin^2 2\theta$$

Finally, use the double angle trig identity provided in the problems statement

$$I_3 = \frac{1}{8} I_0 \left(\frac{1 - \cos 2(2\theta)}{2}\right)$$

$$I_3 = \frac{1}{16} I_0 (1 - \cos 4\theta)$$

$$I_3 = \frac{1}{16} I_0 (1 - \cos 4\omega t)$$

The fraction is thus $f = \frac{1}{16}$.

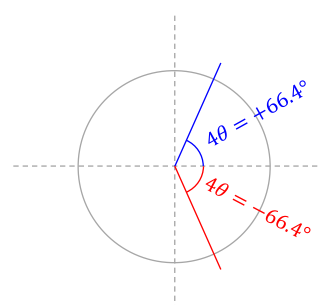
9b) We are told $I_3 = 0.0375 I_0$. Plug in and solve for θ.

$$0.0375 I_0 = \frac{1}{16} I_0 (1 - \cos 4\theta)$$

$$0.600 = 1 - \cos 4\theta$$

$$\cos 4\theta = 1 - 0.600 = 0.400$$

$$4\theta = \cos^{-1}(0.400)$$



Think: cosine inverse gives two results!

$$4\theta = \pm 66.42°$$

$$\boldsymbol{\theta = \pm 16.61°}$$

Strictly speaking, we should see 3.75% transmittance at these angles *during every rotation*.

$$\theta = \pm 16.61° + (360°)n$$

where n is the number of rotations. For the test I am fine with $\theta = \pm 16.61°$.

10a) Total internal reflection (TIR) occurs when travelling from a high index material at an interface with a low index materials. At the critical angle, we assume light is transmitted parallel to the interface ($\theta_2 \approx 90°$).

$$n_{hi} \sin\theta_{critical} = n_{lo} \sin 90°$$

$$\theta_{critical} = \sin^{-1}\left(\frac{n_{lo}}{n_{hi}}\right)$$

$$\theta_{critical} = \sin^{-1}\left(\frac{1}{1.30}\right)$$

$$\boldsymbol{\theta_{critical} = 50.\underline{28}°}$$

At the first interface, the light ray enters a *higher* index material. We expect it should be bent *towards* the normal. Confirm using Snell's law:

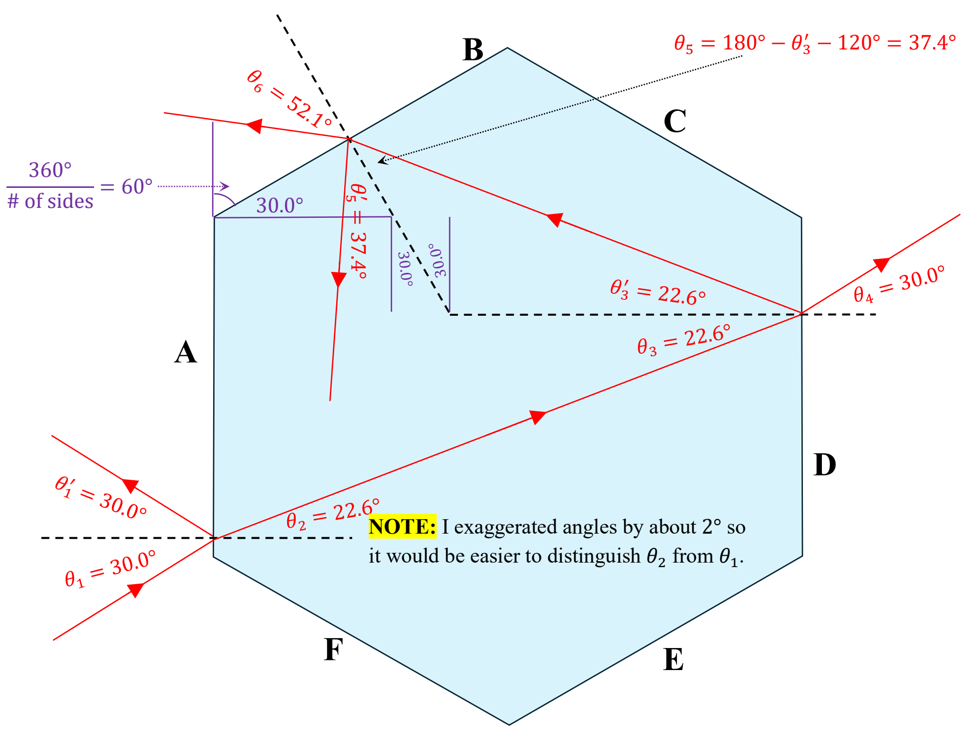
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow \quad \theta_2 = 22.\underline{6}1°$$

I use this information and some geometry to determine the ray hits face D at angle $\theta_3 = 22.\underline{6}1°$.

Notice this angle is *smaller* than the critical angle. TIR does not occur at face **D**.

Some light will be transmitted. I used Snell's law to determine the angle. Notice $\theta_4 = \theta_1$ because those interfaces are parallel.

I used geometry to figure out the angle at the next interface (θ_5). See notes in the figure.
Finally, I notice this angle is also less than the critical angle. Some light will be transmitted.
I used Snell's law to determine $\theta_6 = 52.\underline{1}4°$.



You could go crazy and determine $\theta_7, \theta_8, etc.$ Eventually perhaps you do reach an angle where TIR occurs. To be clear, a small reflection occurs at faces **A** & **D** (97.4% of the light is transmitted). At face **B** another 93.5% is transmitted. To estimate these values I used https://www.lasercalculator.com/fresnel-reflection-and-transmission-calculator/ . Note: a good quick estimate for glass is you lose about 4% at each air glass interface (for incident angles less than 10°). You get more glare (reflection) at larger angles & as the interface indices differ more.

10b) When light enters a new material, energy & frequency are unchanged.

Wavelength in a medium with index n is $\lambda_n = \frac{\lambda_{vacuum}}{n}$. In air, $\lambda_{air} \approx \lambda_{vacuum}$ because $n_{air} \approx 1$. In the ice crystal $\lambda_{ice} = \frac{\lambda_{vacuum}}{1.30} < \lambda_{air}$. Wavelength *decreases* as you go to a higher index material.

Speed in a material is given by $v = \frac{c}{n}$. Speed also decreases as you go to a higher index material.

11a) For the double slit *bright* fringes we know

$$d \sin\theta = \pm m_{double}\lambda \qquad \text{for } m_{double} = 0, 1, 2, 3 \ldots.$$

For single slit *dark* fringes we know

$$a \sin\theta = \pm m_{single}\lambda \qquad \text{for } m_{single} = 1, 2, 3 \ldots.$$

Notice the integers used in these equations do *not* start at the same value! Use the equation sheet carefully!

For the single slit pattern to extinguish the $m_{double} = 2$ bright fringe we require a single slit dark fringe at the same angle as the double slit $m_{double} = 2$ bright fringe.

I will take a ratio of the two equations since $\sin\theta$ & λ will cancel out!

$$\frac{d}{a} = \frac{m_{double}}{m_{single}}$$

$$a = d\frac{m_{single}}{m_{double}}$$

The minimum slit width evidently occurs when $m_{single} = 1$.
Therefore, to extinguish $\underset{bright}{m_{double}} = 2$ we require

$$\boldsymbol{a = \frac{d}{2}}$$

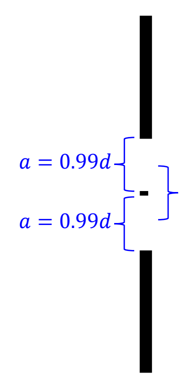
11b) Notice the small angle approximation is not required for this derivation.

11c) If a smaller slit width is used, the dark fringes of the single slit pattern move away from the central max (to larger angles from the central max). At some point, the slit width is small enough such that

$$a \sin 90° = (1)\lambda \qquad \rightarrow \qquad a = \lambda$$

Whenever slit width is smaller than λ, no extinction occurs.
However, as you make the slit widths smaller, less overall light gets through and the overall pattern becomes less intense.

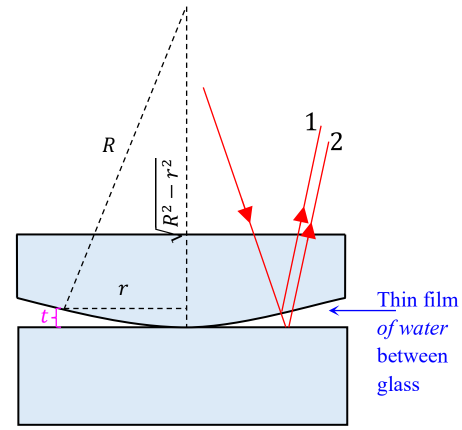
11d) If you increase slit width, the dark fringes of the single slit pattern move towards the central max. However, consider the figure at right showing two slits of width $a = 0.99d$ separated by distance d. If we make slit width $a \geq d$ we no longer have a double slit problem! The pattern would change from a double slit pattern (with some fringes extinguished) to a pure single slit pattern!



What is the point of these last two parts?

Sometimes you cannot simply trust an equation sheet (or the internet) to tell you everything you want to know. In physics, one must think carefully about any limitations on equations used before blindly trusting them…

12a) This is a thin film problem. The thin film is the water-filled gap $(n_{film} = n_{water} = 1.333)$ between the two pieces of glass $n > 1.4$.
If film thickness changes at a constant rate, fringe spacing is constant.
In this scenario, film thickness changes slowly near the center of the lens and rapidly near the edges of the lens. Fringe spacing decreases as the film thickness changes more rapidly.



12b) Ray 1 experiences no phase reversal (reflecting off *lower* index).
Ray 2 experiences a phase reversal (reflecting off *higher* index).
At zero film thickness, near the center of the lens, this implies destructive interference.

12c) The destructive interference condition for a 1 phase reversal scenario is

$$2n_{film}t = m\lambda \qquad \text{for} \quad m = 0, 1, 2, 3, \ldots$$

12d) The film thickness must be described as some function.
Using the figure shown above, I found

$$t = R - \sqrt{R^2 - r^2}$$

If you don't see this, look at workbook problem **35.7** for a more detailed derivation.
The radial position r of dark fringes is found by plugging in this expression and solving for r.

$$2n_{water}\left(R - \sqrt{R^2 - r^2}\right) = m\lambda$$

$$R - \sqrt{R^2 - r^2} = \frac{m\lambda}{2n_{water}}$$

$$\sqrt{R^2 - r^2} = R - \frac{m\lambda}{2n_{water}}$$

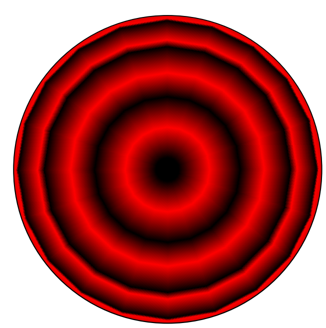
$$R^2 - r^2 = \left(R - \frac{m\lambda}{2n_{water}}\right)^2$$

$$\boldsymbol{r = \sqrt{R^2 - \left(R - \frac{m\lambda}{2n_{water}}\right)^2}}$$

This form is good enough for test day.
Note: since radius is positive, no need to worry about the $\pm$ sign in front of the radical.
Some other styles of writing this answer are shown below.

$$r = R\sqrt{1 - \left(1 - \frac{m\lambda}{2n_{water}R}\right)^2} = R\sqrt{\frac{m\lambda}{n_{water}R} - \left(\frac{m\lambda}{2n_{water}R}\right)^2}$$

Sketch fringe pattern as viewed from the top



12e) To sketch the pattern, refer to the comments made in parts a & b.
The center should be dark (because of a single phase reversal).
The fringes get closer together at larger radii (because film thickness changes more rapidly there).

12f) If you drain the water we still get the same number of phase reversals. The center is still dark.
The term $\left(R - \frac{m\lambda}{2n_{water}}\right)^2 \rightarrow \left(R - \frac{m\lambda}{2}\right)^2$. This term *decreases* causing fringe radius r to *increase*.
Alternatively, realize the effective value of λ increases with the water drained out. The pattern increases in scale.