Pendulums

Apparatus: String, meter sticks, masking tape, protractors, stopwatches, AV stands with T bar on top (for long pendulums)
Number each *table* "Group 1", "Group 2", etc. with a piece of light colored painters tape before class starts.
Students should be in pairs for Part 1. This means each *table* (not each *pair* of tables) needs its own number.

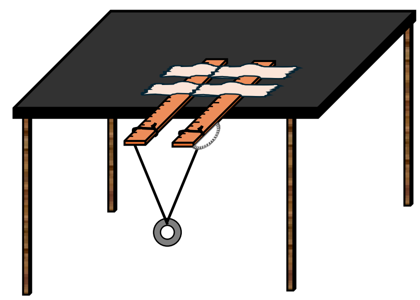
Goals: 1) Determine factors affecting the period of a simple pendulum.
2) Interpret results using various levels of experimental precision.

Before taking any measurements, predict how the following changes will affect the period of a pendulum.

a) Consider *increasing* the pendulum's *mass*. Will the period increase, decrease, or remain the same?
b) Consider *increasing* the pendulum's *length*. Will the period increase, decrease, or remain the same?
c) Consider *increasing* the pendulum's *amplitude*. Will the period increase, decrease, or remain the same?

Physics education research indicates students remember more if they take a guess before doing an experiment.
Note: I just want you to think about these first three questions and formulate an idea in your mind…do not write anything down.

Part 1 – Factors affecting simple pendulum period



Suspend a washer using bifilar suspension as shown at right (figure not to scale).
To make things easier, tape the string to the meterstick instead of tying knots.
Mount a protractor to one ruler for determining the initial angle of the pendulum.

Each table will use a different length pendulum as chosen by their instructor.
For group 1, the washer's center should be hanging 1.00 m below the table.
For group 2, the washer's center should be hanging 0.95 m below the table.
Each successive pendulum should be 5.0 cm shorter than the previous.
Instructor tip: keep the initial pendulum lengths above 50 cm. Restart at 1.00 m once you hit 50 cm.
If I forget to number the tables, ask for your pendulum length *before* starting.

Before timing anything, ensure the pendulum can use a 60.0° start angle (from the vertical) without hitting the table.
Pull the pendulum to a 5.0° starting angle and record the time for three oscillations.

TIP: Say the number zero as you release the pendulum & start the timer.
Count each time the pendulum returns to the initial angle, stopping when you hit three.
Divide the total time for 3 oscillations by 3 to get an experimental value for the period.

Repeat this experiment with the following starting angles: 10.0°, 20.0°, 40.0°, & 60.0°.
Erro $\delta\mathbb{T}$ associated with measuring total time & dividing that by three as well (hint: estimate reaction time).

Using the same length pendulum, add two washers to the pendulum without changing the center of mass height.
You can probably tape on the two additional washers, but make an effort to align all three holes.
Once again, pull the pendulum to a 10.0° starting angle and record the time for three oscillations (& the period).

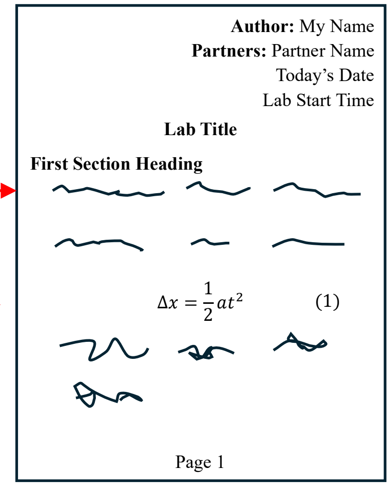
Now make the pendulum half as long as your first pendulum (undo the tape on the ruler and shorten the string).
Remove the extra washers so the mass matches the first experiment (we should only change one variable at a time!).
WATCH OUT! Pendulum length is given by the distance from the table to the washer's center, not string length!
Once again, pull the pendulum to a 10.0° starting angle and record the time for three oscillations (& the period).

I am picky about the order & formatting of your reports because I have to grade many reports quickly.

- Start each section on a new sheet of paper (or lose points).
- Use blank white paper or engineering paper (or lose points).
- Use one side of the paper only (or lose points).
- Double space your work. Why? If I notice you do something incorrectly, I allow you to scratch out the mistake with a single line and write the correction in the space between the lines.
- All conclusion questions (for *all* parts) should be placed at the end of your submission (after data tables and plots).

First page often looks something like this.

Full formatting guidelines (with examples) are found in the Lab Manual Appendices.



I expect each student (not each group) to hand write the answers to these conclusion questions BEFORE going to the next page.

- Most sections of your submission should be in paragraph form (so I can see examples of your writing). HOWEVER, conclusion questions should be answered in a numbered list.
- Answer in complete sentences which make clear what question was asked.
- You do not need to rewrite the questions as long as you paraphrase the question in your answer.

Conclusion Questions for Part 1:

1) Did changing *amplitude* affect the period of oscillation for your simple pendulum? Upon considering errors, are the various periods distinguishable or not?

2) Did changing the *mass* affect the period of oscillation for your simple pendulum? Upon considering errors, are the various periods distinguishable or not?

3) Did changing the *length* affect the period of oscillation for your simple pendulum? Upon considering errors, are the various periods distinguishable or not?

4) Compare the period of the short pendulum at 10.0° to the long pendulum at 10.0°. When the length of the pendulum is doubled, the period is approximately changed by which of the following factors? Hint: take a ratio of the periods (larger length period to shorter length period).

2	$\frac{1}{2}$	$\sqrt{2} \approx 1.41$	$\frac{1}{\sqrt{2}} \approx 0.71$

5) A formula seen in textbooks for the simple pendulum relates period $(\mathbb{T})$ to pendulum length (L) and the magnitude of acceleration due to gravity (g). Use unit analysis to determine which of the following formulas, if any, could be correct. More than one correct answer may be possible.

$\mathbb{T} = 2\pi\frac{L}{g}$	$\mathbb{T} = 2\pi g L$	$\mathbb{T} = \frac{2\pi\sqrt{L}}{g}$	$\mathbb{T} = \frac{2\pi L}{\sqrt{g}}$	$\mathbb{T} = 2\pi\sqrt{\frac{L}{g}}$
$\mathbb{T} = 2\pi\frac{g}{L}$	$\mathbb{T} = \frac{2\pi}{gL}$	$\mathbb{T} = \frac{2\pi\sqrt{g}}{L}$	$\mathbb{T} = \frac{2\pi g}{\sqrt{L}}$	$\mathbb{T} = 2\pi\sqrt{\frac{g}{L}}$

In your write-up, I only want to see the unit check for any dimensionally correct formula(s). If you want, do all the checks on scratch paper then, in your submission, include only the formula(s) which are dimensionally correct.

Part 2 – Reducing Errors by Improving Experimental Design

Use the original length from part 1 (not the shortened length).

The formula frequently found in textbooks (mentioned in question 5 on the previous page) is

$$\mathbb{T}_{th} = 2\pi\sqrt{\frac{L}{g}}$$

Hopefully some of your data agrees with this formula. We'll check that in Conclusion Questions for Part 2.
Notice the above formula shows no dependence on amplitude or mass!

Consider your answer to Question 1 on the previous page.
It is common to find the periods are indistinguishable upon considering measurement errors for all angles.
Why? The error associated with a single trial of this experiment is often somewhat large.

If we repeat trials, is it better to let the pendulum swing more times or use several distinct trials where the pendulum swings three times each? Would it be better to record the period for a single swing many times instead of allowing the oscillation to decay (or would this cause a larger timing error)? Try both ways and see. Does the oscillation decay too rapidly in three trials to get a good estimate of period? What about in 5 or 10 oscillations? Or just two?

Once you dial in the technique you think is best for your particular apparatus, get 10 total computations of the period at 60.0°. Reuse the one data point you've already collected at 60.0° to save time.

Note: for repeated measurements of a single parameter, the experimental value is given by the mean.
The error associated with the average period $\left(\delta\mathbb{T}_{avg}\right)$ is usually found in one of two ways:

1) Use the formula STDEV in Excel to compute the standard deviation. Assume standard deviation is $\delta\mathbb{T}_{avg}$.
2) *If the standard deviation is vanishingly small*, use $\delta\mathbb{T}_{avg} = \frac{\delta\mathbb{T}_{typical\ trial}}{\sqrt{N}}$ where N is the number of trials.

We then write

$$\mathbb{T}_{exp} = \mathbb{T}_{avg} \pm \delta\mathbb{T}_{avg}$$

Before trying to crank out your own error analysis, verify you can reproduce the numbers I did for the Avg, Stdev, and $\delta\mathbb{T}/\sqrt{N}$.

Notice: In this case Stdev is a larger error estimate than $\delta\mathbb{T}/\sqrt{N}$.
Be conservative; use the *larger* of the two as the estimate for $\delta\mathbb{T}_{avg}$.

$$\mathbb{T}_{exp} = \mathbb{T}_{avg} \pm \delta\mathbb{T}_{avg}$$
$$\mathbb{T}_{exp} = 1.62 \pm 0.07 \text{ s}$$

Notice: I round the standard deviation to one sig fig as it is an error calculation.
EXCEPTION: use two digits if the first digit of an error calculation is 1.

Notice: I round $\delta\mathbb{T}/\sqrt{N}$ to one sig fig as it is an error calculation.
EXCEPTION: use two digits if the first digit of an error calculation is 1.

Notice: I use the uncertain column of the error calculation set sig figs for the average.

All trials at 60.0°

Trial	$\mathbb{T}$ (s)	$\delta\mathbb{T}$ (s)
1	1.50	0.07
2	1.55	0.07
3	1.55	0.07
4	1.60	0.07
5	1.60	0.07
6	1.60	0.07
7	1.65	0.07
8	1.65	0.07
9	1.70	0.07
10	1.75	0.07
Avg	1.62	
Stdev	0.07	
$\delta\mathbb{T}/\sqrt{N}$	0.02	

Once you've verified you can reproduce the values shown at right, recompute using your own 10 trials at 60.0°.

Conclusion Questions for Part 2:

6) State the length used for your 10 trials at 60.0°. Include an error estimate ($\mathrm{e.g.}, L = 1.05 \pm 0.02$ m). To be clear, I used $\delta L = 0.02$ m randomly. You must think about how precisely you think you can measure δL in your apparatus with the tools available to you to get your value of δL.

7) Determine the theoretical period associated with this length using

$$\mathbb{T}_{th} = 2\pi\sqrt{\frac{L}{g}}$$

Include an estimate of the error associated with this theoretical value due associated with the uncertainty in the length measurement. Show your work! Include a line with the equation, a line with numbers plugged in, the *unrounded* answer, and the *rounded* answer (with appropriate units). Use

$$\delta\mathbb{T}_{th} = \mathbb{T}_{th}\sqrt{\left(\frac{1}{2}\cdot\frac{\delta L}{L}\right)^2 + \left(\frac{1}{2}\cdot\frac{\delta g}{g}\right)^2}$$

For Santa Maria, CA I'm assuming $g = 9.80 \pm 0.01\,\frac{\mathrm{m}}{\mathrm{s}^2}$ based on an online estimator I found.

8) State the average experimental period you found for your 10 trials at 60.0°. State this answer in the form

$$\mathbb{T}_{exp} = \mathbb{T}_{avg} \pm \delta\mathbb{T}_{avg}$$

9) Using the higher precision results from your ten trials, is $\mathbb{T}_{exp}$ in good agreement with $\mathbb{T}_{th} = 2\pi\sqrt{\frac{L}{g}}$?

Support your result by including a computation of percent difference using

$$\%\text{ difference} = \frac{\text{exp} - \text{th}}{\text{th}} \times 100\%$$

Also include a computation of net percent precision using

$$\%\,precision = \left[\left(\frac{\delta\mathbb{T}_{avg}}{\mathbb{T}_{avg}}\right)^2 + \left(\frac{\delta\mathbb{T}_{th}}{\mathbb{T}_{th}}\right)^2\right]^{1/2} \times 100\%$$

If % difference is significantly larger than % precision, the two are *not* in good agreement.

10) Explain why the formula $\mathbb{T}_{th} = 2\pi\sqrt{\frac{L}{g}}$ does *not* apply to a simple pendulum with a start angle of 60.0°.

11) For large angles, the differential equation describing the simple pendulum's motion becomes non-linear. An exact analytical solution becomes impractical for a student at your level. Use the PhET simulation https://phet.colorado.edu/sims/html/pendulum-lab/latest/pendulum-lab_all.html to generate a better theoretical estimate of the period for your pendulum length. Open the simulation and run it in "Lab mode". Look for a button in the bottom left for "Period Timer". Notice you can change the pendulum length to match the length of your pendulum! Compare your experimental result to the Phet prediction and include a percent difference. Is the percent difference using the PhET lower than the percent difference using the results in question 9?

12) If your pendulum was operated on the moon, would the period increase, decrease, or stay the same? Answer and explain your rationale for credit. Tip: including a relevant equation (and/or calculation) in your answer can really help you explain things.

Submission Checklist

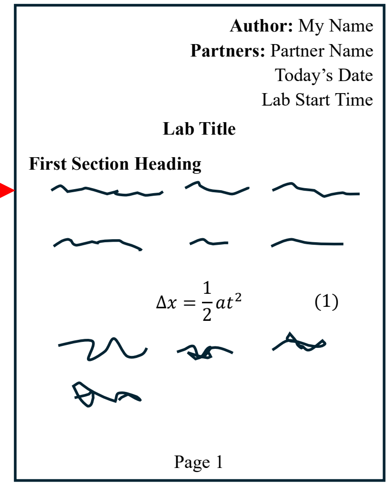
Use blank white paper or engineering paper (or lose points).

Use one side of the paper only (or lose points).

Include the following sections (in this order) in your report this week:

First page should always look something like this.

Full formatting guidelines (with examples) are found in the Lab Manual Appendices.



Procedure (on page 1).

- Use 3rd person.
  - Avoid I, we or you.
  - Avoid commands like "Measure this." That implies "*You* measure this." (2nd person).
- Past tense
  - Avoid "The mass *is* measured". Using "*is* measured" is present perfect (not past tense).
- Use full sentences.
- Do NOT use a bulleted list for this part. I want to see an example of your writing.
- Double space.
  - If I check your work in class and catch a mistake, you can use the blank space for the correction instead of having to erase an entire page and start over.
- Start each section on a new page!
- Use a well-labeled figure approximately a third of a page in size to reduce the amount of writing required. Refer to the figure in your writing. Caption the figure "Figure 1 – Experimental apparatus."
  For this cheesy little lab, your figure might look a lot like mine but with some labels on it.
  You might include both a front view and side view.
  Why? It is probably easiest to label L in a front view and label θ in a side view.
- Write as little as possible while giving a reader enough information to repeat what you did.
  - Include key equipment used (i.e., did you use a stopwatch or a photogate, ruler or calipers).
  - Refer to you figure to make your writing clear & concise.

Data Sheet

- Delete all red text in the spread sheet.
- Hit print preview to ensure it fits on one page.
- Print a single copy and let me look over it for mistakes.
- Fix the mistakes and *then* print one copy for each group member.

Conclusion Questions

- Most sections of your submission should be in paragraph form (so I can see examples of your writing). HOWEVER, conclusion questions should be answered in a numbered list.
- Answer in complete sentences which make clear what question was asked.
- You do not need to rewrite the questions as long as you paraphrase the question in your answer.
- When doing calculations, always show an example of your work. Start by including the algebraic formula, show a step with the numbers plugged in, include the *unrounded* answer, and finally show the *rounded* answer (with appropriate units). Work down the page when showing your work.
- Typically, errors should be rounded to one sig fig.
  - Exception: if the first number of an error calculation is 1, it is common to keep an extra digit.
  - % difference and % precision are considered error calculations. These have no units.
  - Standard deviation ($\sigma_{\mathbb{T}}$) and measurement/propagated errors (e.g., $\delta\mathbb{T}_{exp}$) *with* units!
    The units on $\sigma_{\mathbb{T}}$ & $\delta\mathbb{T}_{exp}$ should match the units of $\mathbb{T}_{exp}$!