Standing String Waves

Apparatus: mechanical oscillators, braided string, mass hangers (50 g), slotted masses for 50 g mass hangers, function generators & power cables, BNC to Banana plug Adapters (Female), banana cables, pulleys from rotation sets, scissors

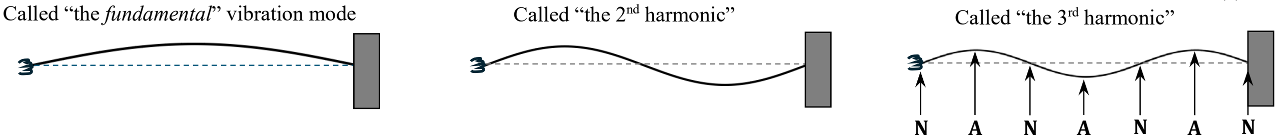
Goal: Use standing waves on a string to investigate relationships between string tension, linear mass density, oscillation frequency, string length, and wave speed.

Theory: For a string fixed at both ends, it is possible to set-up standing waves similar to those shown below.

$\boldsymbol{n = 1}$ **harmonic:** String length $L = 1\left(\frac{\lambda}{2}\right)$
Called "the *fundamental*" vibration mode

$\boldsymbol{n = 2}$ **harmonic:** String length $L = 2\left(\frac{\lambda}{2}\right)$
Called "the 2nd harmonic"

$\boldsymbol{n = 3}$ **harmonic:** String length $L = 3\left(\frac{\lambda}{2}\right)$
Called "the 3rd harmonic"



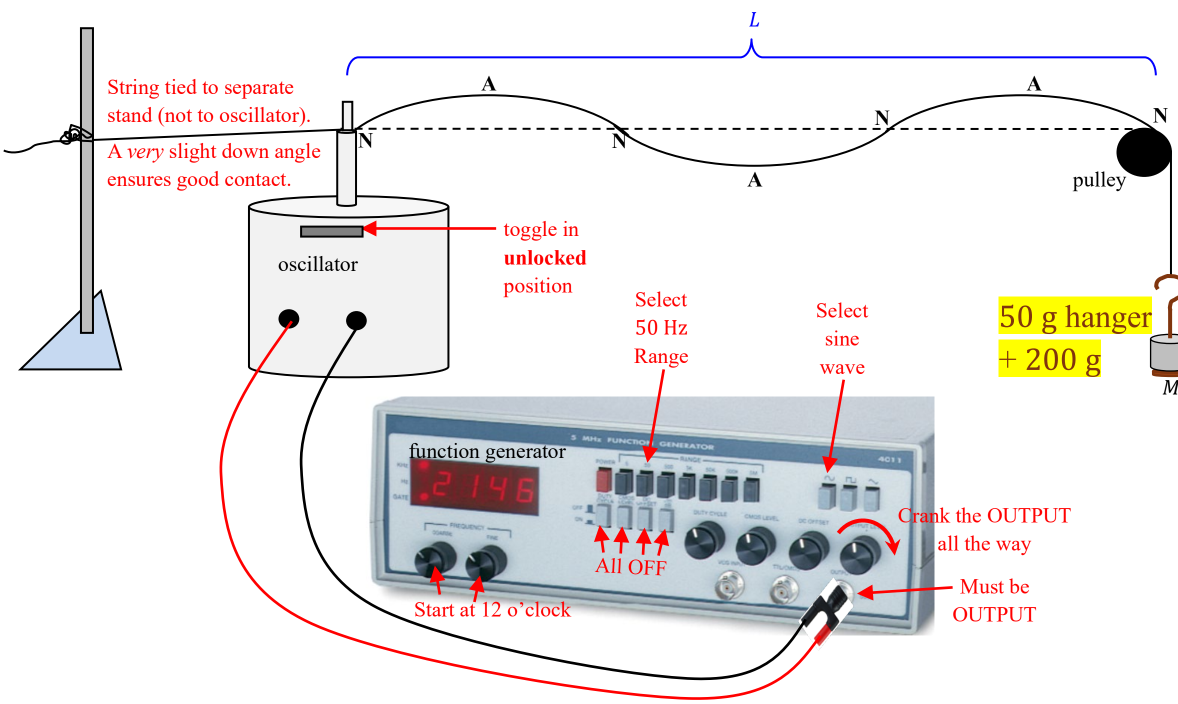
Points on the string which do not move at all called nodes (marked **N** in the rightmost picture).
Points on the string experiencing maximum displacement are called antinodes (marked **A** in the rightmost picture).
The lowest frequency standing wave is called the fundamental frequency (or 1st harmonic).
The higher frequency standing waves are called harmonics.

This experiment will test the above equations in two ways.

1) Fix the tension; change the frequency to find standing waves.
   Record a table of f_n's and n's.
   Plot n vs. f_n and determine an experimental value of wave speed $\left(v_{exp}\right)$ from the slope.
   Compare v_{exp} to the theoretical wave speed given by $v_{th} = \sqrt{\frac{F_{Tension}}{\mu}}$ where μ is linear mass density.
2) Fix the frequency; vary the tension to find standing waves.
   Record a table of m's and n's. Plot $\frac{1}{n^2}$ vs. m to determine an experimental value of g.
   Compare your experimental value to the accepted value near Santa Maria of 9.80 m/s^2.

Apparatus set-up for both parts of the lab

1) Wait to turn on the function generator until you reach that step in the procedure.
2) Determine linear mass density of a piece of string. Cut a long piece of string with length $L_{total} \approx 2.00$ m.
   Use the mass m total length L_{total} to record μ in your data table.
   Use a balance which gives at least 2 sig figs on m!
   Use error propagation formulas to determine sig figs on μ (see *last page* of Lab Manual Appendices).
   Show this calculation in your report.
3) Set-up the apparatus as shown below.
   Adjust the oscillator position until L is between 1.50 m & 1.55 m.
   Ensure the slot on top of the oscillator makes good contact with the string using a tiny down angle.
   Be sure to "unlock" the oscillator.



4) Connect the oscillator to the function generator using the cables and BNC to banana cable adapter.
   Now turn on the function generator. Use a sine wave in the 50 Hz range with no DC offset.
   Start with both COURSE & FINE frequency knobs in the 12 o'clock position.

Part 1

1. Vary the frequency until the string vibrates with only one antinode (the fundamental frequency).
   It may help to estimate this frequency using a theoretical equation so you have a rough idea what f_1 should be.
   Do not assume your experimental result will match perfectly!
   Use the course adjustment to get close then use the fine adjustment to maximize antinode displacement.
2. Record the actual frequencies producing maximum antinode displacement for the harmonics $n = 1$ through 5.
3. Plot the harmonic number versus frequency (n versus f_n).
4. Add a linear trendline to the graph being sure to put the equation and coefficient on the chart.
5. Find the slope of the n versus f_n graph and determine the *experimental* wave velocity from this slope.
   Show your derivation and calculation in your report. Helpful tips regarding the derivation are on the next page.
6. Calculate the *theoretical* wave velocity from the string tension and linear mass density of the string.
7. Calculate the percent difference between two wave velocity calculations for each length.

Part 2

1. Continue to use L between 1.50 m & 1.55 m but set the frequency to $f = 71$ Hz.
2. Calculate the values of m that SHOULD make the string oscillate in the harmonics $n = 1$ through $n = 5$.
3. This time, keep the frequency set at 71 Hz for the entire experiment.
   Instead, *adjust the hanging mass* to cause standing waves to appear.
   Determine the largest and smallest mass (M_{max} and M_{min}) that cause each standing wave to appear.
   The average of these two masses will be your experimental value of M for this part.
   Half the difference will give your estimate for mass $\left(\delta M = \frac{M_{max} - M_{min}}{2}\right)$. Show the calculation in your report.
4. Record the masses required for all harmonics you can find between $n = 1$ and $n = 5$.
   Ignore any harmonics requiring masses of less than 0.050 kg or more than 1.500 kg.
5. Plot M versus $\frac{1}{n^2}$.
6. Add a linear trendline to the graph being sure to put the equation and coefficient on the chart.
7. Find the slope of the line on the graph and determine the *experimental* value of g from this slope.
   Show your work in the calculations section (details mentioned below).
8. Calculate the percent difference between the experimental value and *accepted* value of $g = 9.80\,\frac{\text{m}}{\text{s}^2}$.

TIP: Keep the apparatus set-up until *after* you've tried *all* the conclusion questions.

Derivations for the Calculations section of your report:

To derive theoretical resonant frequencies, I remember the following procedure instead of a formula.

1) Start with $v = f_n \lambda_n$ where v is wave speed, f_n & λ_n are the frequency & wavelength of the n^{th} harmonic.
   Think: for this scenario do you expect $L = n\frac{\lambda_n}{2}$ & $n = 1, 2, 3, \ldots$ or $L = n\frac{\lambda_n}{4}$ & $n = 1, 3, 5, \ldots$?
   Continue until you show
   $$f_n = (some\ expression)n$$
   where "$some\ expression$" involves L & v.

   Now use your experimentally determined slope to determine v_{exp}. Think: $v_{exp} = \frac{slope}{??}$.
   Next, include one line where you plug in numbers without doing any simplification.
   Include another line with the *unrounded* result. Finally show the rounded result with appropriate units.
   Use error analysis to determine δv_{exp} & the appropriate rounding digit for your final result.
   Determine percent precision using techniques similar to our previous labs.

2) Use a nearly identical process as before to show
   $$M = \left(\frac{some\ expression}{g}\right)\frac{1}{n^2}$$
   where M is the hanging mass and "$some\ expression$" involves $\mu, L,$ & f_n.

   Show $g_{exp} = \frac{(some\ expression)}{slope}$ algebraically.
   Next, include one line where you plug in numbers without doing any simplification.
   Include another line with the *unrounded* result. Finally show the rounded result with appropriate units.
   Use error analysis to determine δg & the appropriate rounding digit for your final result.
   Determine percent precision using techniques similar to our previous labs.

Conclusions

1. For Part 1: What was the leading contributor(s) to error (percentage-wise)?
   Include a table showing the percent uncertainty for each *measurement* to support your claim.
   To be clear, include only raw measurements, not parameters calculated from measurements.
   For example, you would include the masses, lengths, and frequencies but not μ, v, or g.
2. For Part 1: Was the experimental result in good agreement with the theory?
   Compare % precision to % difference for Part 1.
   If you aren't sure how to compute % precision, you are supposed to use the error analysis formulas.
   Something similar was done in the previous two weeks of lab as well.
   Try to follow your previous work before asking for help.
   Most of you will probably be able to figure most of it out by now.
3. For Part 1: Under tension the string stretches slightly. If we accounted for the string stretching, should percent difference tend to be more positive or more negative? Explain your rationale thoroughly.
   a. Think: what parameter is affected if the string stretches?
   b. Where does this parameter appear in the computations (in v_{exp} or v_{th})?
   c. Is the affected speed shifted higher or lower by the string stretching?
   d. How does this change propagate through to the percent difference?
4. If you repeated Part 1 with a larger hanging mass, would the observed <u>frequencies</u> to be higher or lower? Explain your reasoning.
5. If you repeated Part 1 with a larger hanging mass, would the observed <u>wavelengths</u> to be larger or smaller? Explain your reasoning.
6. Regarding Part 2: Suppose we wanted to see the maximum number of harmonics possible using hanging masses between 0.050 kg & 1.500 kg. What frequency should be used and how many harmonics would you expect to see. Clearly explain your reasoning. It will help to include calculations to support your rationale.

Submission Checklist:

Title Page (no intro or procedure this week)
Data
Calculations

- For part 1 show calculations of:
  - $\mu, \delta\mu, T, \delta T, v_{th}, \& \ \delta v_{th}$
  - Derivation of determining v_{exp} from the slope & L.
  - $v_{exp}, \delta v_{exp}, \% \ difference, \& \ \% \ precision.$
- For part 2 show calculations of:
  - Derivation of determining g_{exp} from the slope & L.
  - $g_{exp}, \delta g_{exp}, \% \ difference, \& \ \% \ precision.$
- Notice the pattern. If you use an Excel calculation to compute something, you are supposed to show a sample of this calculation in your report.

Conclusions