Wave Optics

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Apparatus: desk lamps, optics bench, laser power supplies, red diode lasers (λ = 670 nm or 650 nm in new models), green diode lasers (λ = 532 nm or 515 nm in new models), scissors, white paper, multiple slit accessory, single slit accessory, diffraction gratings, adjustable lens holder, meter sticks, precision micrometers (if using version A)

Goal: Explore wave properties of light and use wave optics determine the wavelength of a diode lasers.

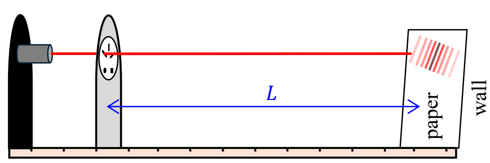
VERSION A:
Choose a diode laser (green or red) and use it for the entire lab.
Do all three experiments using only one color laser.
Write an intro and a procedure *for experiment 2 only*. Remember to use figures to reduce writing!
Think: theory equations go in the intro (3rd person, *present* tense) while experiment specific details go in the procedure (3rd person, *past* tense).
Include data & calculations *for all parts*. Answer *all* conclusion questions.

VERSION B:
Use both the green & red diode lasers.
Do experiments 1 & 3 using both color lasers (four total experiments).
Write an intro and a procedure *for experiment 1 only*. Remember to use figures to reduce writing!
Think: theory equations go in the intro (3rd person, *present* tense) while experiment specific details go in the procedure (3rd person, *past* tense).
Include data & calculations *for all parts*. Answer all conclusion questions *except the last two*.

The last few pages of this manual include large diagrams and theoretical equations for:

- single slits
- double slits
- Babinet's principle
- diffraction gratings
- multiple slit interference

First align the diode laser and slit accessory to the optics bench by doing the following:



1. Place a diode laser on the optics bench about 10 cm apart.
2. Orient the optics bench such that the laser points towards the wall.
3. Ensure the optics bench is perpendicular to the wall.
4. Tape a piece of white paper to the wall such that the laser spot strikes the middle of the paper.
5. Place a slit set 5-10 cm from the laser.
6. Rotate the disk on the slit set until it clicks into place with your desired slit. Start with a double slit.
7. If the laser is not centered on the slit, you will not see a pattern on the paper.
   Adjust laser alignment using the vertical and horizontal adjustment screws.
   If the laser won't align with half turn or so for each knob, ask your instructor for help!
8. Once the laser is align on a double slit, you should be able to rotate the slit accessory disk to try all the different patterns.

Experiment 1 – Use double slit interference pattern to determine λ & compare to manufacturer's specs.

- Select a double slit pattern where it is easy to see at least 12 dark fringes.
- Measure distance L from the slits to the screen.
- Measure the distance between the centers of the two dark fringes on opposite sides of the central max. Divide this measurement by two to determine y for $m = 0$.
- Repeat the measurement of y for fringes $m = 1$ through $m = 5$.
- Create a graph of y vs. m and include a linear trendline (showing R^2 & the trendline equation).
- This training vid walks you through how to make the plot. A checklist for formatting appears on screen.
- Verify the angle θ for the $m = 5$ fringe is less than 10° to ensure the small angle approximation is valid.
- Use the double slit theory page to derive the following *theoretical* expression for y assuming small angles:

$$y = m\frac{L\lambda}{d} + \frac{L\lambda}{2d}$$

  Notice the slope of a plot of y versus m should be $\text{slope} = \frac{L\lambda}{d}$.
- Determine λ_{exp} from the slope of this graph. Verify $\lambda_{exp} = \frac{d}{L} \cdot \text{slope}$.
- Estimate δslope using the LINEST command. Training vid link.
- Determine % precision using
- $\% \text{ precision} = \sqrt{\left(\frac{\delta\text{slope}}{\text{slope}}\right)^2 + \left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta d}{d}\right)^2} \times 100\%$
- The figure at right gives manufacturer's specs on the slits.
- Determine % difference $= \frac{exp - th}{th} \times 100\%$ where the theory value is the manufacturer's specification. See the apparatus notes at the top of page 1 for these values.

Specifications

Slit width tolerance	±0.005 mm
Slit spacing tolerance	±0.01 mm *or* ±0.005 mm if spacing is 0.125 mm

Experiment 2 – Determine the width of a hair using a single slit interference pattern.

Use single slit theory to determine the relationship between y & m where a is slit *width* (not slit *spacing* d).
Assume the small angle approximation is valid but verify this is true for your highest order fringe.
Use your theoretical equation to relate the slope to $a, \lambda,$ & L.
Rearrange this relationship to determine an expression for a in terms of slope, $\lambda,$ & L.

Babinet's principle implies the diffraction pattern from a hair should produce the same pattern as a single slit.
In this situation we can shift slit width a to hair diameter D!
Shine the laser beam on a single strand of hair instead of a single slit.
I suspect you could tape the hair to the adjustable lens holder a few centimeters from the beam.
I did see a video where someone taped the hair onto the laser itself (in the center of the beam)…no clue if that works better.

Collect data for at least 5 dark fringes.
Use a plot of y versus m to determine the slope & ultimately the hair diameter D.
Use a high precision micrometer to directly measure the hair's diameter.
Since both values of D come from experimental measurements, let us agree to call D_{exp} the value obtained from the slope of the graph and D_{th} the value obtained from direct measurement with a high precision caliper.
Determine the % precision using the following formula and assuming $\delta\lambda = 1$ nm.

$$\% \text{ precision} = \sqrt{\left(\frac{\delta\text{slope}}{\text{slope}}\right)^2 + \left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta\lambda}{\lambda}\right)^2} \times 100\%$$

Experiment 3 – Determine $\frac{\textbf{\# lines}}{\textbf{cm}}$ on a diffraction grating.

For the grating, we measure distances from the central max to interference *maxima* (*bright* fringes).
WATCH OUT! The small angle approximation is typically NOT valid for grating experiments.

You will need to mount the diffraction grating in an adjustable lens holder (don't touch the surface of the grating).
The grating will split the laser beam vertically or horizontally (depending on how you orient the grating).
With the grating oriented horizontally, tape a meter stick to the wall and read the m values directly from the ruler.
You only need to record values of y for $m = 1$ & 2.
Use diffraction grating theory to determine $n = \frac{\textbf{\# lines}}{\textbf{cm}}$.
Instead of making a graph, use your derived formula to determine two values of n and average them.
Compare your result to the stated value of n with a percent difference.
For a theoretical value, use the manufacturer's stated value (typically printed on the grating itself).

If you want to derive the error analysis, let me know in lab. It requires ugly partial derivatives so I used Wolfram Alpha to do the heavy lifting for me. Knowing how to use a program like Wolfram Alpha is a good skill so I encourage you to ask. All that said, for % precision I found

$$\begin{matrix}\text{\% precision of single} \\ \text{measurement of } n\end{matrix} = \sqrt{\left[\frac{\delta y}{y} \cdot \frac{1}{1 + \left(\frac{y}{L}\right)^2}\right]^2 + \left(\frac{\delta \lambda}{\lambda}\right)^2} \times 100\%$$

Exploration 1 – How does increasing the number of slits affect the diffraction pattern?

Without taking any data, look at the patterns formed by the multiple slits. Compare to multi-slit theory.

Exploration 2 – Compare single slit to single line.

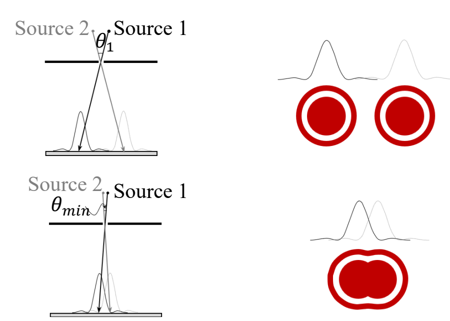
Without taking data, compare the pattern formed by a single slit to that formed by a single line (of the same size).

Exploration 3 – Observe patterns formed by circular apertures.

Without taking data, observe the pattern formed by a circular aperture. Try a different size if available.

I suppose we could try setting up two lasers far from the aperture, both aimed at the same aperture. Perhaps we could try to test the minimum resolution criteria

$$\theta_{min} = \frac{1.22\lambda}{d}$$



Exploration 4 – Observe patterns formed by hexagonal and square apertures.

Just try it out and have fun!

Conclusions:

1. Which experiment (double slit, or grating) most precisely determines the wavelength of the laser?
2. How should the double slit pattern change if the slit separation is *decreased* (check your answer)? Specifically discuss the fringe spacing!
3. The old version of the lab had $\lambda_{red} = 670$ nm but new documentation from Pasco indicates $\lambda_{red} = 650$ nm. Which specified wavelength should have fringes closer together? Explain your rationale.
4. Our physics lab is often very cold. How will abnormally cold room temperature affect the interference and diffraction patterns (compared to doing the experiment in a significantly warmer room? Explain for credit.
5. Describe the similarities and differences between the single slit and double slit patterns. Specifically discuss the width of the central maximum in each pattern. You might include a sketch of each pattern to clarify your result. Doing workbook problem **36.12 part a** (super quick pattern identification question) may help you here.

Manufacturer's specifications:

Single Slit Set

- 4 single slits (slit widths 0.02, 0.04, 0.08, 0.16 mm)
- 1 variable slit (slit width varies from 0.02 to 0.20 mm)
- 1 square pattern
- 1 hexagonal pattern
- 1 random opaque dot pattern (dot diameter = 0.06 mm)
- 1 random hole pattern (hole diameter = 0.06 mm)
- 1 opaque line of width 0.08 mm
- 1 slit/line comparison, line and slit have similar width (0.04 mm)
- 2 circular apertures (diameters 0.2 mm and 0.4 mm)

Specifications

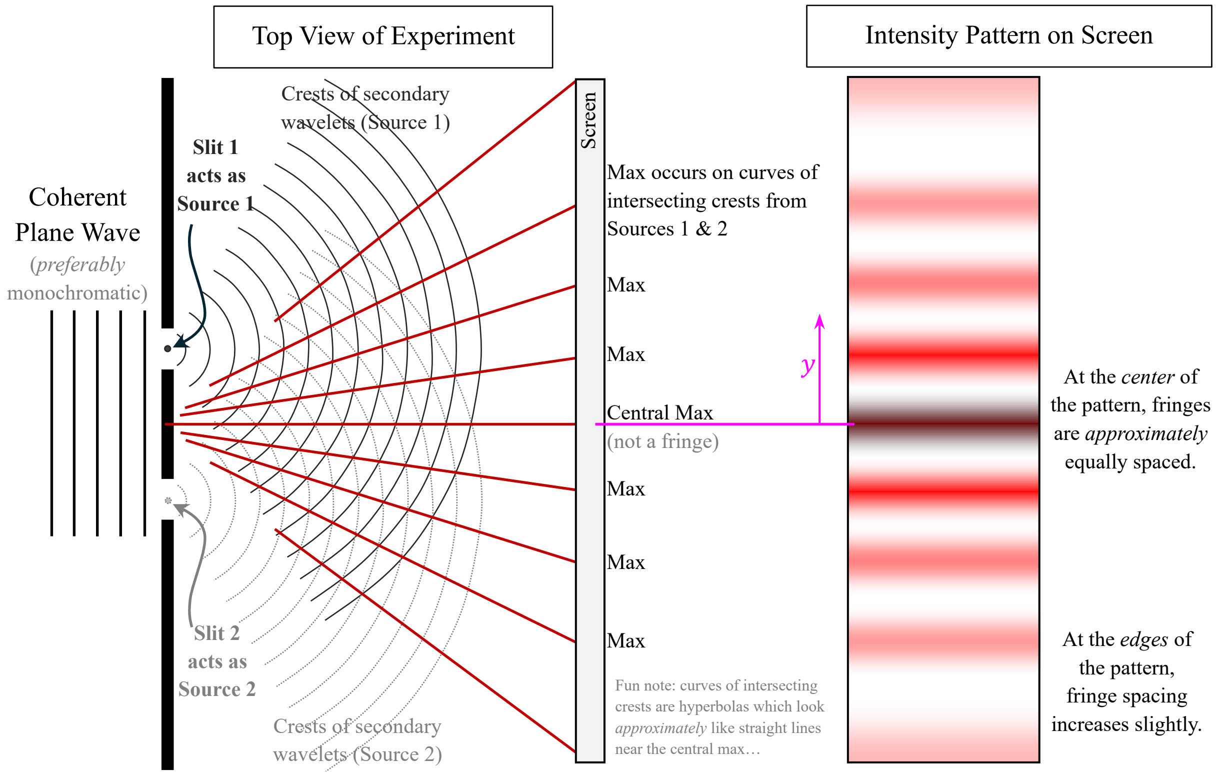
Slit width tolerance	±0.005 mm
Slit spacing tolerance	±0.01 mm *or* ±0.005 mm if spacing is 0.125 mm

Multiple Slit Set

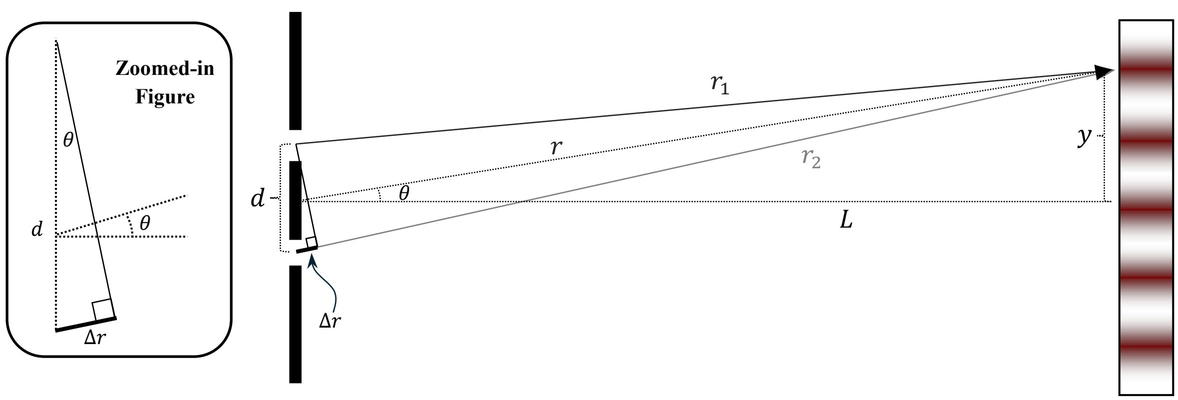
- 4 double slits (slit width/separation in mm: 0.04/0.25, 0.04/0.50, 0.08/0.25, 0.08/0.50)
- 1 variable double slit (slit separation varies from 0.125 to 0.75 mm with constant slit width of 0.04 mm)
- 4 comparisons: single/double slit with same slit width(0.04 mm)
- double/double slit with same slit width (0.04 mm), variable separation (0.25 mm to 0.50mm)
- double/double slit with different slit widths (0.04, 0.08 mm), same separation (0.25 mm)
- double/triple slit with same slit width (0.04 mm), same separation (0.125 mm)
- set of 4 multiple slits (2, 3, 4, 5 slits) with same slit width (0.04 mm), same separation (0.125 mm)

Young's Double Slit Experiment

In Young's original experiment, sunlight passed through a tiny hole produced coherent plane waves of light.
In modern experiments, typically a laser produces coherent plane waves of light.
By using a laser with a single wavelength (*monochromatic* light), the patterns are easier to interpret.



Typical Approximations: Screen far from Slits, Restrict to Fringes near the Center



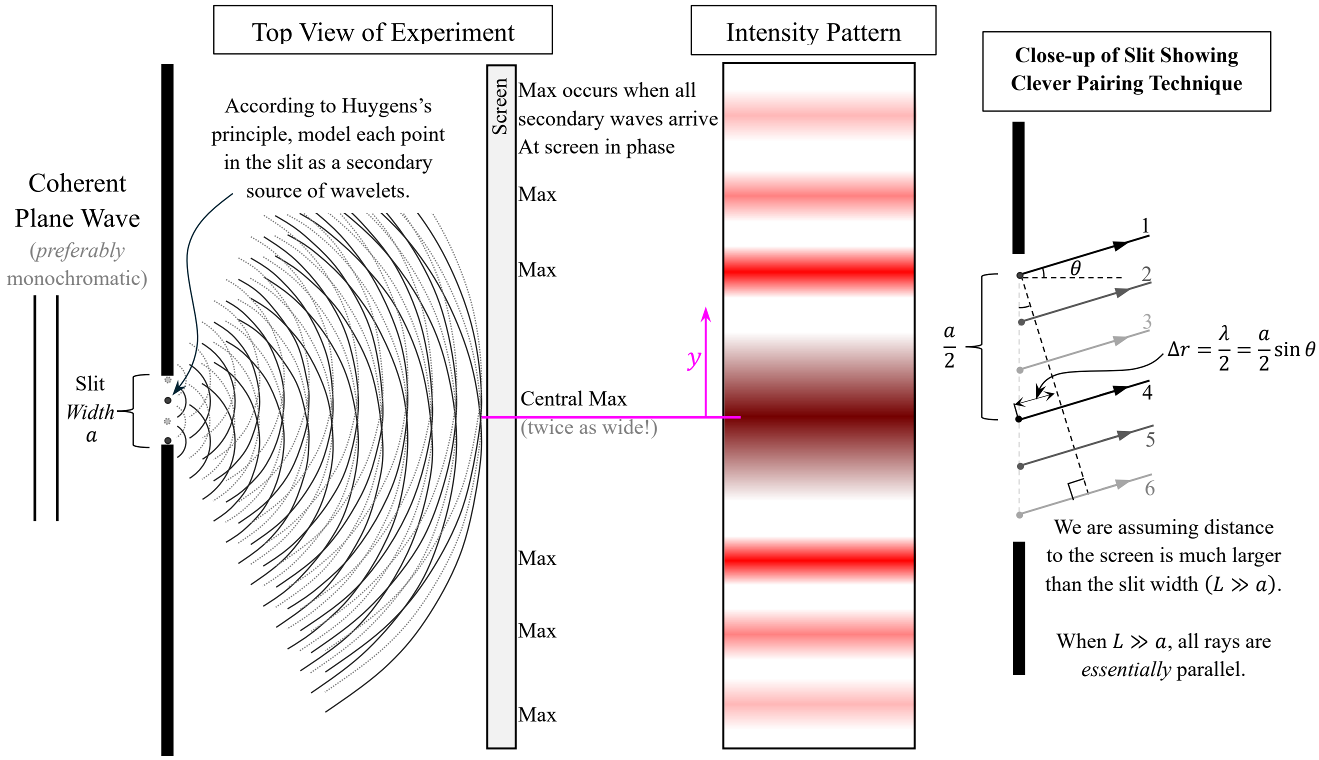
Double Slit Equations

$\Delta r = \pm m\lambda$ for $m = 0, 1, 2, \ldots$ (*constructive* interference, *bright* fringes)	$\Delta r = d\sin\theta$ (assumes $L \gg d$)
$\Delta r = \pm\left(m + \frac{1}{2}\right)\lambda$ for $m = 0, 1, 2, \ldots$ (*destructive* interference, *dark* fringes)	$\sin\theta \approx \tan\theta = \frac{y}{L}$ (assumes $L \gg y$)
Intensity Pattern: $I = 4I_0\cos^2\frac{\phi}{2}$ where $\phi = \frac{2\pi d}{\lambda}\sin\theta$ (I_0 is intensity from 1 slit at screen)	$\Delta r \approx \frac{yd}{L}$ (assumes $L \gg d$ & $L \gg y$)

*The intensity pattern is also affected by *single slit diffraction*. This is discussed in the *next* chapter.
**The intensity pattern equation given already assumes $L \gg d$.

Single Slit Interference Pattern

An interference pattern arises from a single slit as well (historically called a diffraction pattern).
When light passes a sharp edge (or through a slit), the waves spread out and can interfere with each other.



Consider the *rightmost* figure above showing six secondary wavelets produced according to Huygens's principle.
For this particular choice of angle, the path length difference between rays 1 & 4 is exactly $\frac{\lambda}{2}$.
The wavelet sources for rays 1 & 4 are separated by exactly half of the slit width. Using some trig, on finds

$$\Delta r = \frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

Because the path length difference is exactly $\frac{\lambda}{2}$, these wavelets destructively interfere.
Similar logic applies to pairs 2 & 5 as well as 3 & 6. At this angle, any secondary wavelet source cancels this way!

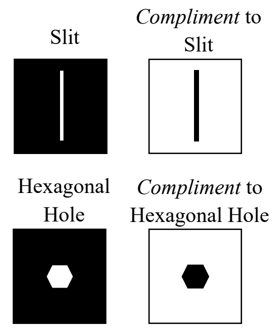
Single Slit Equations

$a\sin\theta = \pm m\lambda \quad \text{for } m = 1, 2, \ldots$ (*destructive* interference, *dark* fringes) (assumes $L \gg a$)	$\sin\theta \approx \tan\theta = \frac{y}{L}$ (assumes $L \gg y$)	$I = I_{max}\left[\frac{\sin\beta}{\beta}\right]^2 \quad \text{where} \quad \beta = \frac{\pi a}{\lambda}\sin\theta$ I_{max} is the maximum intensity level of the central maximum. **WATCH OUT!** This equation assumes units of radians on β!

- We use a for the *slit width* of a single slit and d for the *slit separation* between double slits.
- A single slit uses $a\sin\theta_{dark} = \pm m\lambda$ (*destructive*) while a double slit uses $d\sin\theta_{bright} = \pm m\lambda$ (*constructive*).
  The extra $\frac{\lambda}{2}$ comes from the clever pairing trick which required us to cut the single slit in half.
- Notice the single slit equations start numbering at $m = 1$ while double slit equations start with $m = 0$!
- Remember: the central maximum is not called a fringe.
- Bright fringe locations are *approximately* (not *exactly*) given by $a\sin\theta = \pm\left(m + \frac{1}{2}\right)\lambda$ for $m = 1, 2, \ldots$
  To find *exact* bright fringe locations, differentiate the intensity equation with respect to position.
- If a is approximately equal to (or less than) λ, diffraction occurs but you won't see fringes!
- Diffraction always occurs, but it is probably most noticeable when a is on the order of about 10λ or less.
- The sinc function (sine cardinal $= \operatorname{sinc} x = \frac{\sin x}{x}$) appears frequently in digital signal processing & Fourier transforms.

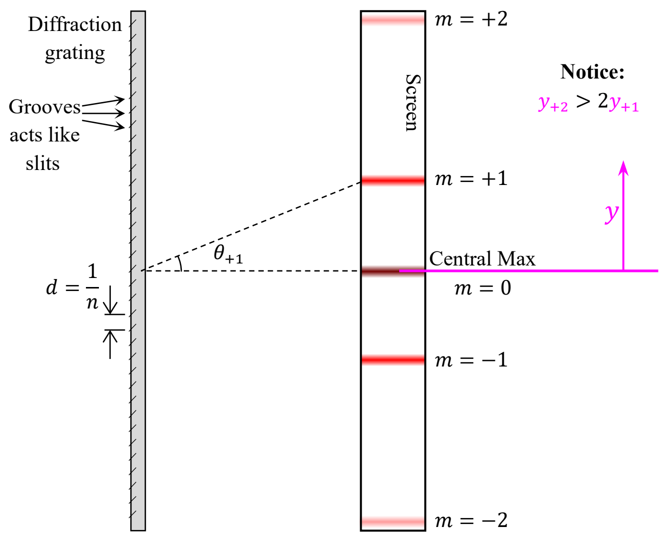
Diffraction from Complimentary Patterns

Babinet's Principle states the diffraction pattern from complimentary objects is nearly identical. In particular, the *shape* of the diffraction pattern remains the same but the *intensity* may differ. Examples of complimentary objects are shown at right. Notice that the diffraction pattern from a thin wire (or human hair) should produce the same diffraction pattern as a single slit (with different intensity).



Diffraction gratings

A *diffraction grating* is usually a piece of glass with an enormous number of grooves etched in it. The grooves in the glass produce an interference pattern similar to the multi-slit interference patterns shown at right. Because the effective number of slits is so large, we typically ignore the intermediate maxima and focus solely on the primary maxima.



Diffraction Grating Terms & Equations

n = number of lines *per unit length* **WATCH OUT!** To do calculations, convert units to $\frac{\text{lines}}{\text{meter}}$	$d = \frac{1}{n}$ **WATCH OUT!** First convert units of n to $\frac{\text{lines}}{\text{meter}}$	$d \sin\theta_{bright} = \pm m\lambda$

Note: In the real world, the grating & incident light are often angled. As a result, the diffraction pattern is not symmetric. The fringes on one side of the pattern are angled more than the other side. The diffraction equation becomes more complicated.
If you end up doing spectroscopy work, you may end up learning about the more general equations at that time. For now, let's keep it simple…

Multi-slit Interference Patterns

At right I show multi-slit interference patterns for various number of slits.

I generated these plots using $\lambda = 500$ nm, $I_0 = 100\,\frac{\mathrm{W}}{\mathrm{m}^2}$, & $d = 70.0$ μm.

NOTE: For these plots I assumed slit width (a) was smaller than λ.

This allowed me to ignore the extinction of minima due to single slit diffraction.

Notice several important aspects of increasing the number of slits:

- Not all maxima have equal intensity.
- For every value of N, we see primary maxima peaks whenever $d \sin\theta_{bright} = m\lambda$. In this set of plots, this occurs when $\theta_{bright} \approx 0.4°$.
- As N increases, intensity of the primary maxima increases as N^2.
- As N increases, the primary maxima get narrower.
- The number of minima between the adjacent primary maxima is $N - 1$.
- In the real world, the central max is brightest and other primary maxima will be less intense due to single slit diffraction effects.