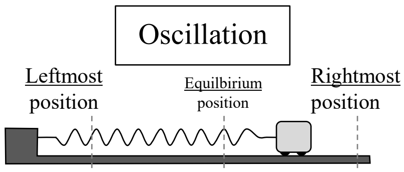
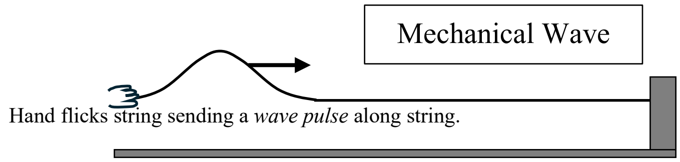
Introduction to Waves (page 23 in Physics Workbook Volume 3)
Just like oscillations, waves are everywhere in nature.
At first glance, the equations we use to analyze waves will probably seem similar.
It is important to realize waves *relate* to oscillations, but they are not the same.

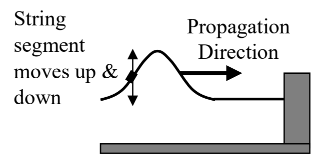


Energy remains, *on average*,
at the equilibrium position.



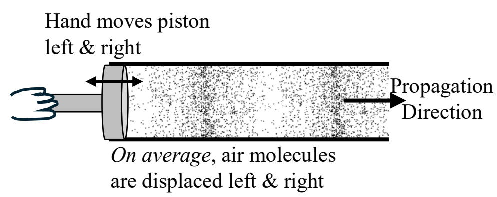
Energy travels from one place to another.
Individual segments of the string oscillate up & down.

Transverse versus Longitudinal Waves (page 23 in Physics Workbook Volume 3)
For our string wave, individual string segments oscillate up & down while wave energy travels left to right. We say the displacements of elements in the medium are perpendicular to the direction wave propagation.



Waves with displacements ⊥ to propagation direction are called *transverse* waves.

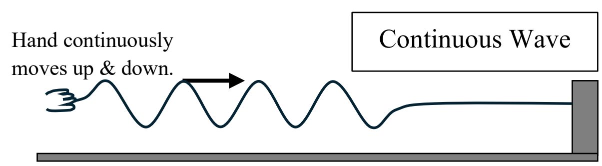
Now imagine a piston in a pipe being pushed back and forth.
The air molecules are moving all over the place. On average, however, the air molecules oscillate back and forth in the *same* direction as the piston.
Wave energy propagates down the pipe (to the right in this case).



Waves with displacements ∥ to propagation direction are called *longitudinal* waves.

Continuous Sinusoidal Waves (page 25)
Now assume the hand wiggles the string continuously.
Ignore what happens when the string hits the wall (for now).
The hand motion is sinusoidal generating the string shape

$$\boldsymbol{y(x,t) = A\sin(kx - \omega t + \phi)}$$



Dan Russell has some amazing visualizations for sound waves here:
https://www.acs.psu.edu/drussell/demos.html
This link is appropriate right now: https://www.acs.psu.edu/drussell/Demos/waves-intro/waves-intro.html

There is also a solid tutorial walking through basic wave physics with a bunch of great simulations from oPhysics.
It helps so much to see the animations instead of still images: https://ophysics.com/w.html
Wave tutorial: https://ophysics.com/waves1.html

Superposition Principle & Wave Interference (page 30)

The *superposition principle* says

$$y_{total}(x,t) = y_1(x,t) + y_2(x,t) + y_3(x,t) + \cdots$$

What does this mean?

> If more than one wave travels on a string at the same time, sum the equations modelling those waves to describe the displacement of string elements caused by those waves.

Said another way (that isn't quite phrased properly):

> The superposition principle says you can add up all the waves to get the total wave.

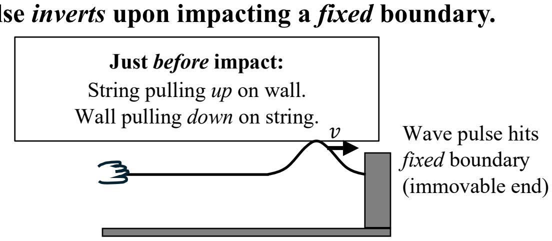
This superposition principle holds for all waves, not just string waves.

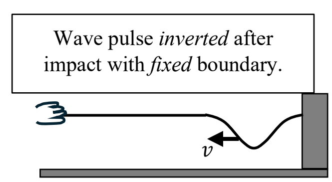
- To get the overall shape of the string, add the two wave pulse shapes together at any instant in time.
$$y_{total}(x,t) = y_1(x,t) + y_2(x,t)$$
- *Constructive interference* – Places where vertical position of both waves have the same sign experience an *increase* in the overall wave height.
- *Destructive interference* – Places where vertical position of both waves have opposite signs experience a *decrease* in the overall wave height.

Wave Reflections (page 34)

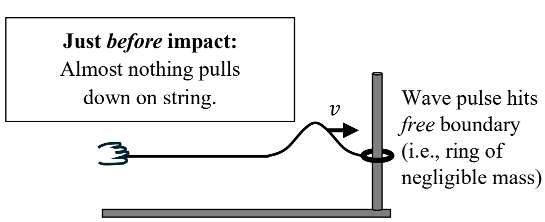
cool websites: https://www.animations.physics.unsw.edu.au/jw/waves_superposition_reflection.htm
https://www.acs.psu.edu/drussell/demos/reflect/reflect.html

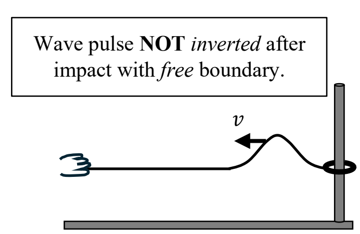
Consider a string is tied to something heavy at one end (i.e., a brick).
A wave pulse impacting the brick will not lift the brick up.
We call this a *fixed* end (or fixed boundary).
A wave pulse *inverts* upon impacting a *fixed* boundary.





Now imagine the string secured to a post by a ring of negligible mass.
The ring is free to slide up and down with negligible friction.
We call this a *free* end (or free boundary).
A wave pulse does NOT *invert* upon impacting a *free* boundary.





Standing Waves (page 35)

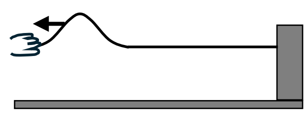
Cool website: https://www.acs.psu.edu/drussell/Demos/SWR/SWR.html

Consider a sinusoidal travelling wave incident upon a boundary (fixed or free).
The reflected wave interferes with the incident wave and, under certain conditions, a standing wave is created.
The conditions are affected by the total number of inversions due to reflections.
Standing waves create a striking visual pattern where it almost appears that no wave is travelling at all!
Various points on the string oscillate up and down…but each point oscillates a different amount.
You have to see a simulation or demo to get it….no book can ever do this phenomena justice.

Results for Standing Waves with <u>Matched</u> or <u>Mixed</u> Boundary Conditions

Typically, matched boundary conditions on strings implies both ends of the string are *fixed*.
Even though the hand may be wiggling up & down to create waves, it is a fixed boundary.
Because the hand inverts incident waves, we must consider it as a fixed boundary.

Waves invert upon hitting hand. Think of hand as *fixed* boundary.



Resonant frequencies – frequencies which create standing waves.
Node – A point on the string with *zero* vertical motion at resonance.
Antinode – A point on the string with *maximum* vertical motion at resonance.

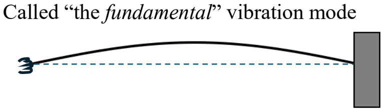
- Adjacent *nodes* (**N**'s) are separated distance $\frac{\lambda}{2}$.
- Adjacent *antinodes* (**A**'s) are also separated distance $\frac{\lambda}{2}$.
- The distance between a node and an antinode is $\frac{\lambda}{4}$.

n = Harmonic Number

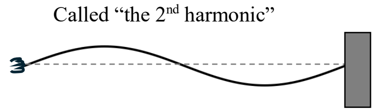
For <u>*matched*</u> *boundary conditions*, harmonic number corresponds to the # of *half*-wavelengths on the string.

- $n\left(\frac{\lambda}{2}\right) = L$ where $n = 1, 2, 3, \ldots$ for *matched* boundary conditions $\left(\lambda_n = \frac{2L}{n}\right)$ where $n = 1, 2, 3, \ldots$

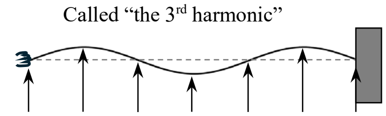
$n = 1$ harmonic: String length $L = 1\left(\frac{\lambda}{2}\right)$
Called "the *fundamental*" vibration mode



$n = 2$ harmonic: String length $L = 2\left(\frac{\lambda}{2}\right)$
Called "the 2nd harmonic"



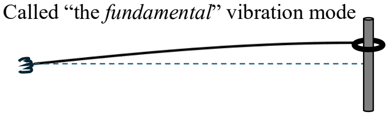
$n = 3$ harmonic: String length $L = 3\left(\frac{\lambda}{2}\right)$
Called "the 3rd harmonic"



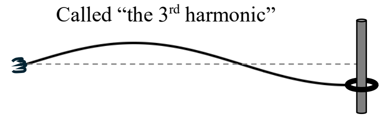
For <u>*mixed*</u> *boundary conditions*, harmonic number corresponds to the # of *quarter*-wavelengths!

- $n\left(\frac{\lambda}{4}\right) = L$ where $n = 1, 3, 5, \ldots$ for *mixed* boundary conditions $\left(\lambda_n = \frac{4L}{n}\right)$ where $n = 1, 3, 5, \ldots$
- Notice *only odd harmonics* appear when you have *mixed* boundary conditions!

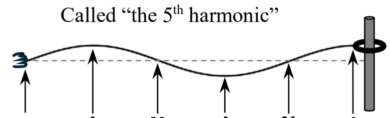
$n = 1$ harmonic: String length $L = 1\left(\frac{\lambda}{4}\right)$
Called "the *fundamental*" vibration mode



$n = 3$ harmonic: String length $L = 3\left(\frac{\lambda}{4}\right)$
Called "the 3rd harmonic"



$n = 5$ harmonic: String length $L = 5\left(\frac{\lambda}{4}\right)$
Called "the 5th harmonic"

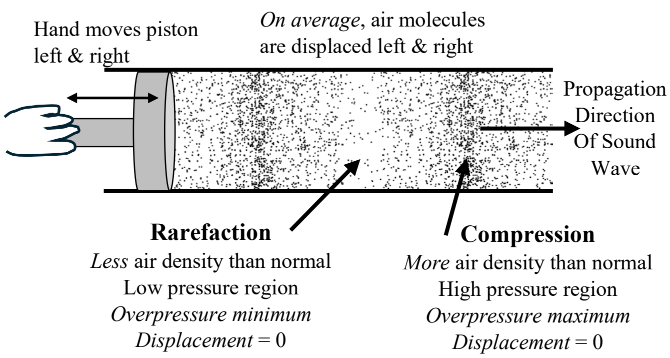


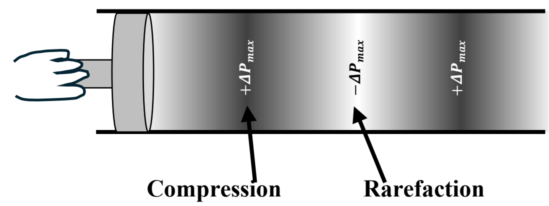
To derive resonant frequencies: I remember the following procedure instead of a formula.

- We know $v = f_n\lambda_n$ and $v = \sqrt{\frac{F_{Tension}}{\mu}}$ where $\mu = \frac{\text{mass of string}}{\text{length of string}}$ = string linear mass density.
- <u>*Matched Boundaries*</u>: $L = n\frac{\lambda_n}{2}$ & $n = 1, 2, 3, \ldots$ <u>*Mixed Boundaries*</u>: $L = n\frac{\lambda_n}{4}$ & $n = 1, 3, 5, \ldots$
- Solve for f_n.

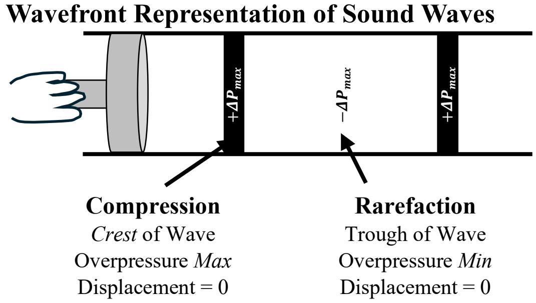
Sound Waves (page 45)
We assume humans with good hearing can hear frequencies ranging *approximately* from 20 Hz to 20 kHz.
If you try to understand this page without looking at simulations, you are doing yourself a disservice.
Dan Russell visualizations: https://www.acs.psu.edu/drussell/Demos/phase-p-u-sine/phase-p-u-sine.html
oPhysics animations: https://ophysics.com/w6.html

Compressions & Rarefactions Representation of Sound Wave



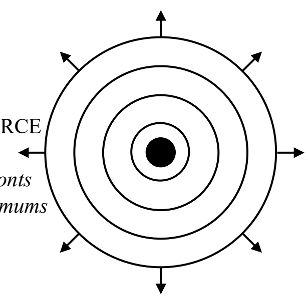


Wavefront Representation of Sound Waves



Spherical Wave
Energy spreads out *spherically*
Solid Black circle at center = SOURCE

Concentric rings are called *wavefronts*
Wavefronts are *overpressure maximums*



Sound Wave Equations in Overpressure & Displacement Forms

Displacement Form	Overpressure Form
$s(x,t) = s_{max}\,\mathbf{cos}(kx - \omega t)$ s_{max} = air molecules displaced maximum amount from previous equilibrium location $\pm s_{max}$ occurs halfway between rarefactions & compressions	$\Delta P(x,t) = \Delta P_{max}\,\mathbf{sin}(kx - \omega t)$ ΔP_{max} = maximum CHANGE from normal atmospheric pressure $\pm \Delta P_{max}$ occurs at compressions & rarefactions
WATCH OUT! For this chapter I usually set phase angle $\phi \to 0$. In the real world one might need to include a phase angle.	
$\Delta P_{max} = v_{sound}\rho\omega s_{max}$ **Units:** $\frac{\text{m}}{\text{s}} \times \frac{\text{kg}}{\text{m}^3} \times \frac{\text{rad}}{\text{s}} \times \text{m} = \frac{\text{kg}}{\text{m}\cdot\text{s}^2} = \frac{\text{N}}{\text{m}^2} = \text{Pascal} = \text{Pa}$	

$$v_{sound} = c = \text{speed of sound in materials (i.e., metals, water, etc.)} = \sqrt{\frac{Bulk\ Modulus}{density}} = \sqrt{\frac{B}{\rho}}$$

$$v_{\substack{sound \\ air}} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma R(273.15\ ^\circ\text{C} + T_C)}{M}} \qquad \text{we typically assume } v_{\substack{sound \\ air}} \approx 331.4\,\frac{\text{m}}{\text{s}} + \left(0.61\,\frac{\text{m}}{\text{s}\cdot{}^\circ\text{C}}\right) T_C$$

WATCH OUT! Many factors affect $v_{s\,air}$ including temperature, humidity, and elevation. This approximation is decent (to within a few %) for air temperatures between -100 °C to $+100$ °C. Unless otherwise specified, assume $v_{\substack{sound \\ air}} = 343\,\frac{\text{m}}{\text{s}}$ and $\rho_{air} \approx 1.2\,\frac{\text{kg}}{\text{m}^3}$.

Standing Sound Waves in Pipes (page 53)
You would be crazy not to look at some animations for these:

- Dan Russell https://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html
- Shows reflection at open end of tube: https://newt.phys.unsw.edu.au/jw/flutes.v.clarinets.html

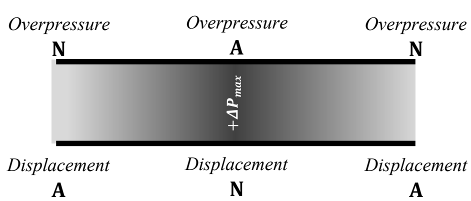
To derive resonant frequencies: I remember the following procedure instead of a formula.

- We know $v_{sound} = f_n \lambda_n$
- *Matched* Boundaries: $L = n\frac{\lambda_n}{2}$ for $n = 1, 2, 3, \ldots$ *Mixed* Boundaries: $L = n\frac{\lambda_n}{4}$ for $n = 1, 3, 5, \ldots$
- Solve for f_n.

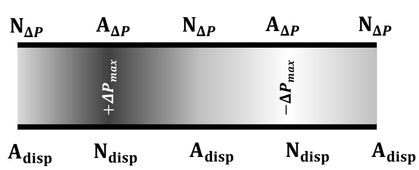
***Displacement* Node** – A point in the air column causing zero *displacement* at resonance.
***Displacement* Antinode** – A point with maximum *displacement* (positive *or negative*) at resonance.
***Overpressure* Node** – A point in the air column causing zero *overpressure* at resonance.
***Overpressure* Antinode** – A point in the air column causing maximum (or minimum) *overpressure* at resonance.

- The distance between a node and an antinode (of the same type) is $\frac{\lambda}{4}$.
- A *displacement node* is an *overpressure antinode* (and vice versa).

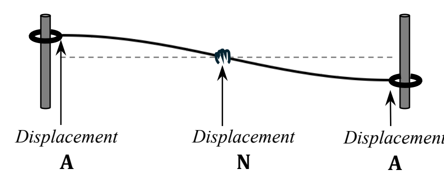
Example of *Matched* Boundary Conditions
Pipe *open* at both ends

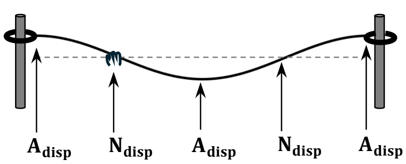


Pressure antinodes flip between $\pm\Delta P_{max}$ every $\mathbb{T}/2$!



Analogous String Wave
String free at both ends, oscillator at nodes in between

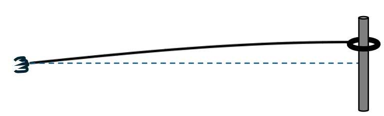


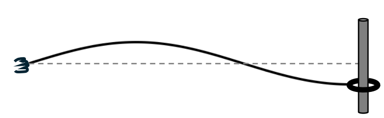


Example of *Mixed* Boundary Conditions
Pipe closed at one end, open at other end

Pressure antinodes flip between $\pm\Delta P_{max}$ every $\mathbb{T}/2$!

Analogous String Wave
Oscillator at one end, string free at other end





WATCH OUT! The reflection from an open-ended tube actually happens *slightly outside* the tube.
The *effective* length of the tube is increased slightly by the open end as follows:

$$\Delta L_{open} = 0.6 \times (tube\ radius) = 0.3 \times (tube\ diameter)$$

If the tube is *open at both ends*, you would need to *add this correction for both ends*.
On exam days I will ignore this correction but in lab assignments you should account for it.