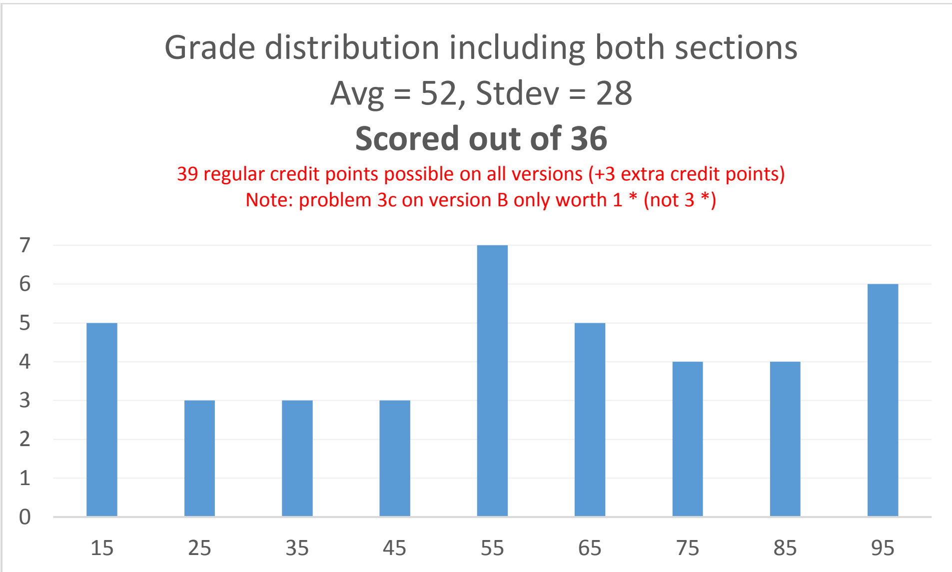
163fa21t1aSoln (YELLOW TEST)

Solutions begin on the next page.
Distribution of grades shown on this page.



1a) Point charges moving around we typically use:

$$K_i + U_i = K_f + U_f$$

Point charge configurations energy is

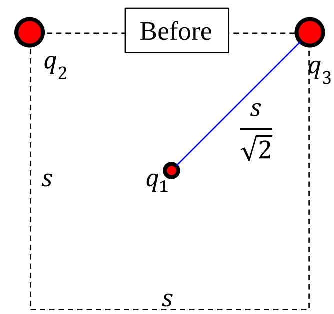
$$U_{tot} = U_{12} + U_{13} + U_{23}$$

Initial state:

$$U_i = U_{12i} + U_{13i} + U_{23i}$$

$$U_i = \frac{kq_1q_2}{r_{12i}} + \frac{kq_1q_3}{r_{13i}} + \frac{kq_2q_3}{r_{23i}}$$

$$U_i = \frac{kq_1q_2}{\frac{s}{\sqrt{2}}} + \frac{kq_1q_3}{\frac{s}{\sqrt{2}}} + \frac{kq_2q_3}{s}$$



Watch out! Most common mistake is people forget $U_f \neq 0$!!!

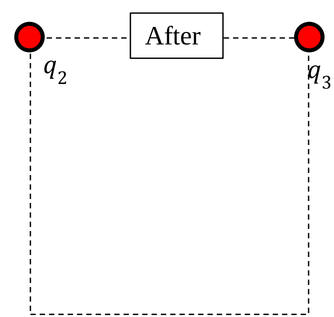
Consider the final state shown in the bottom figure at right…

$$U_f = U_{12f} + U_{13f} + U_{23f}$$

$$U_f = \frac{kq_1q_2}{r_{12f}} + \frac{kq_1q_3}{r_{13f}} + \frac{kq_2q_3}{r_{23f}}$$

$$U_f = \frac{kq_1q_2}{\infty} + \frac{kq_1q_3}{\infty} + \frac{kq_2q_3}{s}$$

$$U_f = \frac{kq_2q_3}{s}$$



I found

$$m = 16\sqrt{2}\frac{ke^2}{sv^2}$$

$$\boldsymbol{m \approx 22.6\frac{ke^2}{sv^2}}$$

1b) When analyzing this scenario, I find it easier to start from

$$\Delta K = -\Delta U$$

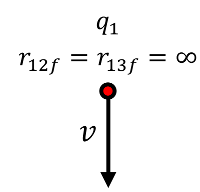
The potential energy associated with electric fields is independent of mass.
Therefore q_1 experiences the same ΔU for this scenario.

$$K_f - K_i = -\Delta U$$

$$K_f - 0 = -\Delta U$$

$$\frac{1}{2}mv_f^2 = -\Delta U$$

$$v_f = \sqrt{-\frac{2\Delta U}{m}}$$



Doubling mass *reduces* speed by factor $\sqrt{2}$. Consider a ratio:

$$\frac{v_f'}{v_f} = \frac{\sqrt{-\frac{2\Delta U}{(2m)}}}{\sqrt{-\frac{2\Delta U}{m}}} = \frac{1}{\sqrt{2}}$$

2) I was slightly sneaky here.
I hope you noticed I never defined the constant *k*!
You were expected to notice that and add a line of code.

The way I would code it might look something like what is shown below. There are many correct ways to do this question…

Note: the names of the variables are unimportant as long as the code does what it is supposed. On test day, short variable names are probably recommended.
Hopefully you'd use longer variable names in a real code to make it easier for anyone forced to read the code.

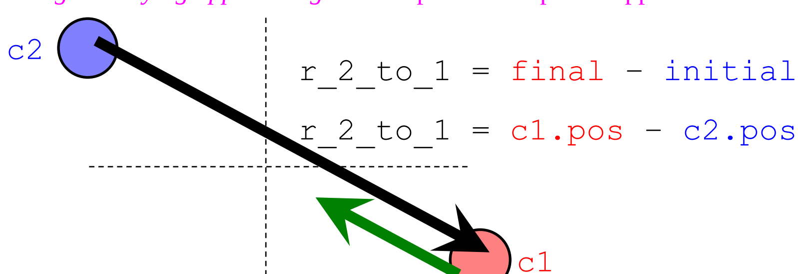
```
1  GlowScript 3.1 VPython
2  #assume SI units (kg, m, C) on all numbers
3
4  c1 = sphere( color=vec(1, 1, 0), radius=0.5 )
5  c1.pos = vec(3.2415,-1.254,9.743015)
6  c1.q = 2.5749186543245e-6        #charge 1
7  c1.m = 1e-7                      #mass 1
8
9  c2 = sphere( color=vec(0, 1, 1), radius=0.5 )
10 c2.pos = vec(-2.45678,1.23456,9.87654)
11 c2.q = -1.35765987456e-6         #charge 2
12 c2.m = 1e-2                      #mass 2
```

Line 13	`k = 8.99e9`
Line 14	`r_2_to_1 = c1.pos - c2.pos  #see figure below`
Line 15	`Force_on_1 = k*c1.q*c2.q*r/mag(r)**3`
Line 16	`acceleration_of_1 = Force_on_1/c1.m`
Line 17	`print(acceleration_of_1)`

Many people screw up by flipping the terms in **Line 14**.
Recall the definition of the Coulomb force we've been using is

$$\vec{F}_{2\text{ on }1} = \frac{k q_1 q_2 \vec{r}_{2\text{ to }1}}{r^3_{2\text{ to }1}}$$

WATCH OUT! Notice the charges *in this case* carry opposite signs.
When you plug in the charge values *for this case* the result for force will be a negative factor times $\vec{r}_{2\text{ to }1}$.
For cases with charges carrying opposite signs we expects force points opposite the direction of $\vec{r}_{2\text{ to }1}$!




```
Force_on_1 = k*c1.q*c2.q*r/mag(r)**3
Force_on_1 = (negative #)*r
Force_on_1 points opposite the direction of r
```

3a)

$$[\rho] = \frac{[\alpha]}{[r]}$$

$$[\alpha] = \frac{\mathrm{C}}{\mathrm{m}^2}$$

Be aware

$$\alpha \neq \frac{\mathrm{C}}{\mathrm{m}^2}$$

The constant α is a number with units (if & when you would eventually plug in a number).
When we write $[\alpha]$ that implies "the units of α".
Use the rectangular brackets appropriately or expect to lose points.

3b) Gauss's Law gives us

$$EA_{Gaussian} = \frac{q_{in}}{\epsilon_0}$$

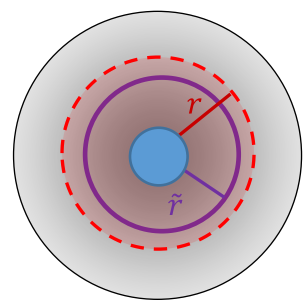
Since we were asked to compute the field (mag) inside the surface, we are only enclosing SOME of the charge.

The upper limit of integration should NOT go all the way to the outer radius of the object!!!

$$EA_{Gaussian} = \frac{1}{\epsilon_0}\int_R^r dq_{shells}$$

Think: what you are doing when you do the integral is summing up the charge of every shell inside the Gaussian surface. As you move radially outwards from the center, charge density of each shell to decreases according to the given formula for ρ. This is why we CANNOT use the tricks associated with uniform distributions for this problem…

$$EA_{Gaussian} = \frac{1}{\epsilon_0}\int_R^r \rho_{\text{at shell radius}}\, dV_{shell}$$



$$E(4\pi r^2) = \frac{1}{\epsilon_0}\int_R^r \frac{\alpha}{\tilde{r}}\, 4\pi\tilde{r}^2\, d\tilde{r}$$

WATCH OUT! The shell radius $\tilde{r}$ and the Gaussian radius r are not the same thing!
The shell radius can take on *any* value between the lower limit & upper limits…
From there either of the following answers is fine with me:

$$E = \frac{\alpha}{\epsilon_0 r^2}\left(\frac{r^2}{2} - \frac{R^2}{2}\right) = \frac{2\pi k\alpha}{r^2}(r^2 - R^2)$$

Note: in the question I asked for electric field (not electric field magnitude).
In this problem, most physicists assume a positive result implies radially outwards.
As such, we typically leave off the $\hat{r}$ in the answer (but explicitly writing it is also correct).

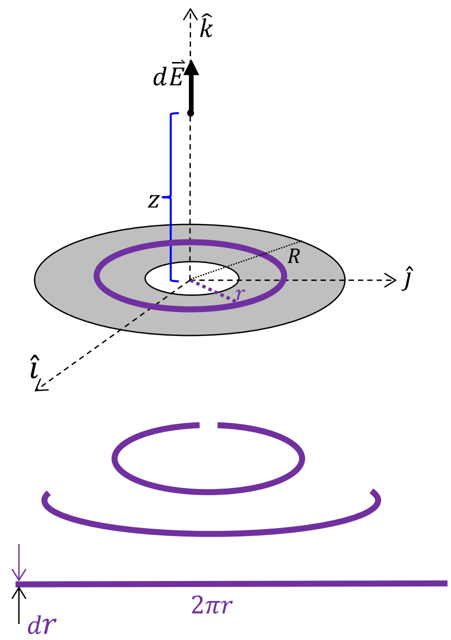
3c) Only worth 1 point. On some tests I forgot update from the old problem (***) to what was actually given (*).
The above result is valid when $r = 3R$. Plug in those values and solve for α to find:

$$\alpha = \frac{9}{4}E\epsilon_0 = \frac{9E}{16\pi k}$$

4) We tend to build washers and disks from rings.
The process is nearly identical to the process shown in 22.21 & 22.22 in the workbook.

$$\sigma = \frac{total\ charge}{total\ area} = \frac{Q}{\pi R_2^2 - \pi R_1^2}$$

$$dq = \sigma\, dA = \frac{Q}{\pi(R_2^2 - R_1^2)}(2\pi r)dr$$



The integral is

$$\vec{E} = \int d\vec{E}$$

$$\vec{E} = \int \frac{k\, dq\, z}{((\text{ring radius})^2 + z^2)^{\frac{3}{2}}}\hat{k}$$

$$\vec{E} = \int \frac{k\, \sigma\, dA\, z}{((\text{ring radius})^2 + z^2)^{\frac{3}{2}}}\hat{k}$$

$$\vec{E} = \int \frac{k \left\{\frac{Q}{\pi(R_2^2 - R_1^2)}(2\pi r)dr\right\} z}{((\text{ring radius})^2 + z^2)^{\frac{3}{2}}}\hat{k}$$

$$\vec{E} = \frac{2kQz\hat{k}}{R_2^2 - R_1^2}\int_{R_1}^{R_2} \frac{r}{(r^2 + z^2)^{3/2}}dr$$

$$\vec{E} = \frac{2kQz\hat{k}}{R_2^2 - R_1^2}\left[-\frac{1}{(r^2 + z^2)^{\frac{1}{2}}}\right]_{R_1}^{R_2}$$

Eliminate the minus sign by flipping the limits…

$$\vec{E} = \frac{2kQz\hat{k}}{R_2^2 - R_1^2}\left[\frac{1}{\sqrt{r^2 + z^2}}\right]_{R_2}^{R_1}$$

Final result for field.

$$\vec{E} = \frac{2kQz}{R_2^2 - R_1^2}\left[\frac{1}{\sqrt{R_1^2 + z^2}} - \frac{1}{\sqrt{R_2^2 + z^2}}\right]\ \hat{k}$$

Comments on this result:

- I asked for field (not field magnitude), you answer must include the unit vector $\hat{k}$
- Because $R_2 > R_1$ we expect the first term in brackets should be larger than the second. Result points $+\hat{k}$ as expected for a positively charged washer or disc.

Note: we expect this result should become $\frac{kQ}{z^2}\hat{k}$ in the limit $z \to \infty$. In this case it is NOT obvious.
Showing how this is done is described in some workbook problem (**22.22** I believe?).
Perhaps you can see an application of the binomial expansion coming into play???

5a) A negative charge would require the left plate at higher potential.
A positive charge would require the right plate to be held at higher potential.
We were given no indication of the sign of the charge on the ball.
As such, it is **impossible to determine** which plate must be held at higher potential.

WAIT A MINUTE…I THOUGHT YOU TOLD US:
"Assume q is positive unless you are told otherwise."
Yes, I did say that all the time.
HOWEVER, if you listened closely, I generally followed that up with something like:
"If the charge is negative we can always flip the sign when we plug in values."

If you don't believe me, check the video lectures…

Finally, students were specifically told we did not know the sign of the charge during the exam.
Again, we were given no indication of the sign of the charge on the ball.
As such, it is **impossible to determine** which plate must be held at higher potential.

5b) Even though we don't know the *signs* of the charges or plates, we still know the *sizes* of the electric field & electric force experienced by the ball.

$$\text{field } magnitude = E_{\| plates} = \frac{|\Delta V|}{s}$$

$$F = (\text{charge } magnitude) \times (\text{field } magnitude)$$

$$F = \frac{|q|\,|\Delta V|}{s}$$

From there I did an FBD (see far right figure). In equilibrium $a_x = a_y = 0$.

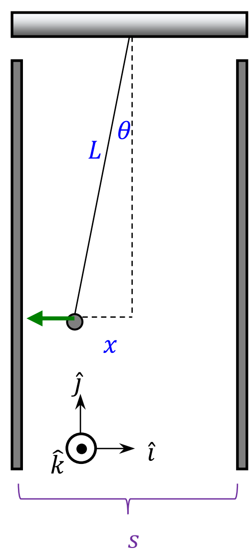
$$\Sigma F_x\text{:}\quad T\sin\theta = F \qquad \Sigma F_y\text{:}\quad T\cos\theta = mg$$

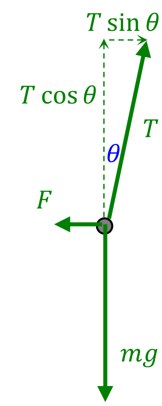
A ratio of horizontal & vertical force eqt'ns eliminates T and produces $\tan\theta$.
We were told the small angle approximation applies.
In this case this informs us

$$\tan\theta \approx \sin\theta = \frac{x}{L} \quad \textbf{or} \quad \tan\theta = \frac{x}{\sqrt{L^2 - x^2}}$$

Do some algebra to find

$$|\boldsymbol{\Delta V}| = \frac{\boldsymbol{mgsx}}{|\boldsymbol{q}|\sqrt{\boldsymbol{L^2 - x^2}}} \approx \frac{\boldsymbol{mgsx}}{|\boldsymbol{q}|\boldsymbol{L}}$$





NOTICE: Since the sign of the charge is ambiguous in the problem statement, I expect you to add in the clarification of the absolute value sign for charge. I *may* be lenient on the absolute value on charge if that caused you to miss the earlier part…

5c) The electric field is constant regardless of the ball's position within the plates.
This assumes the ball is far from the ends of the plates (an assumption stated in the problem).
Since the electric field is unchanged, we expect force $F = qE$ will also remain unchanged.
The ball's weight is also unchanged.
This implies the angle is also unchanged (consult the FBD)!!!
If the angle is unchanged, as L increases parameter x must also increase!

6a) The following equations have an implied $\hat{\imath}$ on each side. A negative results implies a force to the left.

$$F_x = qE_x$$

$$F_x = (-e)(-slope)$$

$$F_x = (e)(slope)$$

$$F_x = (1.602 \times 10^{-19}\ \text{C})\left(\frac{-40.0\ \text{nV}}{2.00\ \mu\text{m}}\right)$$

$$\boldsymbol{F_x = -3.20 \times 10^{-21}\ \text{N}}$$

Force MAGNTIUDE is absolute value of this number!!!

THINK: Our analogy tells us to think of the electron as a bubble below the surface of the curve.
If released from rest we would expect the bubble to move to a more negative horizontal position.
Negative result for F_x (force to the *left*) makes sense.

6b) **To the left.**
Note: A bubble under the curve would move up and to the left.
HOWEVER, the analogy uses only the *horizontal* position of the bubble as it floats along under the curve.

6c) If released from rest, the electron *initially* has zero kinetic energy.
It does have *initial* potential energy given by

$$U_i = qV_i$$

$$U_i = (-1.602 \times 10^{-19}\ \text{C})(10.0\ \text{nV})$$

As the electron travels to the left, it eventually reaches a horizontal position with equal potential.
When this occurs, we know the electron must once again have all potential energy (no kinetic energy).
This must be the point where the electron reverse direction!
This occurs when $V_f = 10.0$ nV **at horizontal position $\boldsymbol{x_f = -3.00\ \mu\text{m}}$.**

7a) We know the *static* electric field is zero inside a conductor.
The image below is my attempt to indicate the pattern of charge density by using increasing line thickness...
The inner surface of each conductor is polarized in such a way as to balance the NET charge *inside that particular conductor*.
Net charge *on any one shell* is still Q as dictated by the problem statement.
As such, the charge at each radius of interest is tabulated at right.

Radius	Charge	Charge Density
1	$-Q$	$\sigma_1 = -\frac{Q}{4\pi R_1^2}$
2	$+2Q$	$\sigma_2 = +\frac{Q}{2\pi R_2^2}$
3	$-2Q$	$\sigma_3 = -\frac{Q}{2\pi R_3^2}$
4	$+3Q$	$\sigma_4 = +\frac{3Q}{4\pi R_4^2}$
5	$-3Q$	$\sigma_5 = -\frac{3Q}{4\pi R_5^2}$
6	$+4Q$	$\sigma_6 = +\frac{4Q}{4\pi R_6^2}$

7b) A Gaussian surface drawn with radius between R_4 & R_5 is shown with a dotted pink line below. Notice it encloses net charge $\boldsymbol{+3Q}$. All charge on the outer conductor combines in such a way as to cause no NET contribution to the electric field for radii less than R_5!!!

Since we are outside of a spherical surface, we know

$$E = \frac{kq_{in}}{r^2} = \frac{3kQ}{r^2}$$

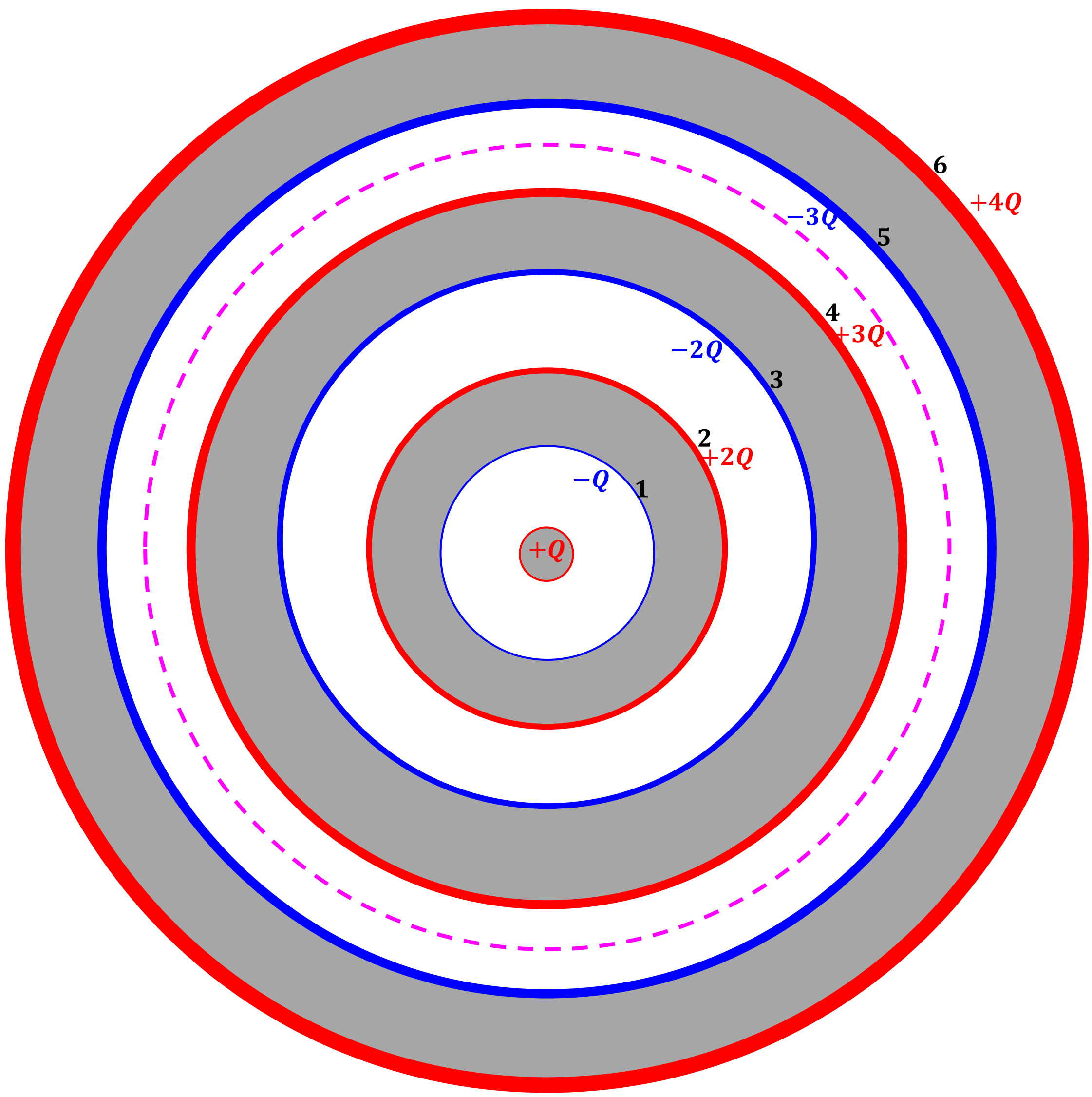
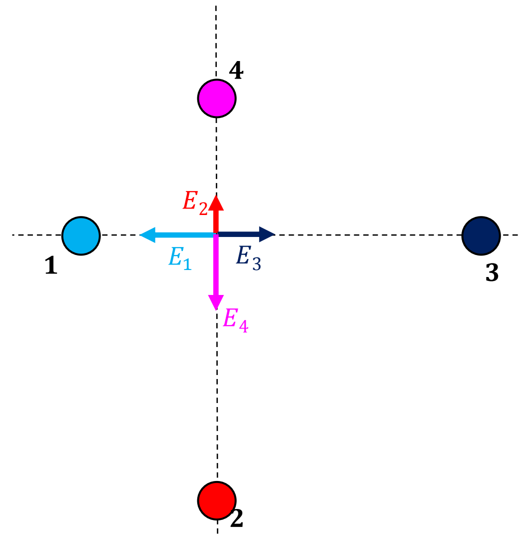


Figure at right shows charges and field contribution from each charge.
Distances and field arrows drawn to scale.
Charges are listed below.

- $q_1 = -e$
- $q_2 = +2e$
- $q_3 = -3e$
- $q_4 = +e$



8a) **Net electric potential is *negative*.**

$$V_{tot} = V_1 + V_2 + V_3 + V_4$$

$$V_{tot} = -\frac{ke}{d} + \frac{k(2e)}{2d} - \frac{k(3e)}{2d} + \frac{ke}{d}$$

8b) Twice the charge does not balance twice the distance!
Unlike electric potential calculations, *distance is squared in the electric field calculation*.
Net field points DOWN & LEFT.

8c) You should find

$\vec{E}_1 = \frac{ke}{d^2}(-\hat{\imath})$	$\vec{E}_2 = \frac{k(2e)}{(2d)^2}(+\hat{\jmath})$	$\vec{E}_3 = \frac{k(3e)}{(2d)^2}(+\hat{\imath})$	$\vec{E}_4 = \frac{ke}{d^2}(-\hat{\jmath})$

$$\vec{E}_{NET} = \frac{ke}{d^2}\left[-\frac{1}{4}\hat{\imath} - \frac{1}{2}\hat{\jmath}\right]$$

Sketch this field arrow and use inverse tangent to find

$$\boldsymbol{\theta = -116.6° \; from \text{ the positive horizontal axis}}$$

Note: if you answered $-116.6°$ *below* the positive horizontal axis you are wrong. The word below operates like a negative sign in that sentence.

This is why it is best to sketch the arrow to clearly explain things whenever possible (i.e. if you had to present this data to other people for some reason).

Extra credit solutions are not written up.
Writing these solutions takes days of work as it is…