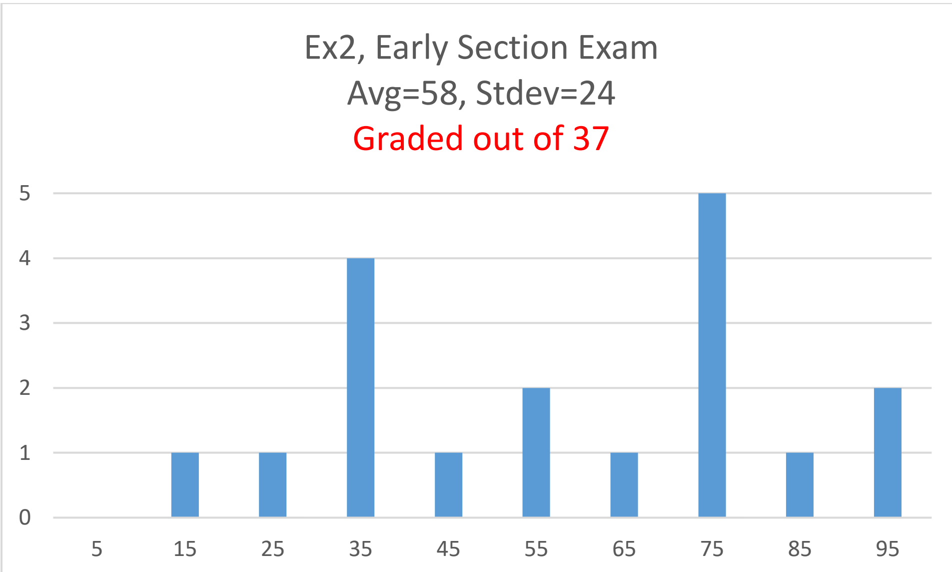
161fall21t2aSoln

YELLOW TEST

Distribution on this page. Solutions begin on the next page.



**1) For the capacitance of each group in parallel, just add them up. Then use

$$C_{eq} = \left(\frac{1}{C} + \frac{1}{C+2C} + \frac{1}{3C} + \frac{1}{4C}\right)^{-1}$$
$$C_{eq} = C\left(1 + \frac{1}{3} + \frac{1}{3} + \frac{1}{4}\right)^{-1}$$
$$\boldsymbol{C_{eq} \approx 0.522C}$$

2abc) $\boldsymbol{B_x = 0}$ $\boldsymbol{B_y > 0}$ $\boldsymbol{B_z = indeterminable}$

Explanation:

Magnetic force is given by

$$\vec{F} = q\vec{v} \times \vec{B}_{ext}$$
$$F(-\hat{\imath}) = qv(-\hat{k}) \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right)$$

Cancelling minus signs…

$$F(\hat{\imath}) = (-|q|)v(\hat{k}) \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right)$$

From this we see B_x *must* equal zero. If $B_x \neq 0$, the force would have a $\hat{\jmath}$ component (contradicting problem statement)!

We see $B_y > 0$. We know this because $\hat{k} \times \hat{\jmath} = -\hat{\imath}$…TIMES THE NEGATIVE CHARGE!
We *require* a *positive* value for B_y!

Finally, we see the information in the problem statement is insufficient to determine anything about B_z.
No matter what value you plug in for B_z it will drop out in the calculation since $\hat{k} \times \hat{k} = 0$.

3ab) I found current in resistor R was

$$\boldsymbol{i = \frac{7\mathcal{E}}{11R}}$$

which produced power

$$\mathcal{P} = i^2R$$
$$\mathcal{P} = \frac{49}{121} \cdot \frac{\mathcal{E}^2}{R}$$
$$\boldsymbol{\mathcal{P} \approx 0.405\frac{\mathcal{E}^2}{R}}$$

I will do my best to grade your set of equations based on how you chose to label currents, loops, & directions…
Usually there is no partial credit on 3b…

4a) Capacitance 2 decreases when the dielectric is removed. Total capacitance also decreases.

4b) Initially, C_2 stores more charge than C_1.
After the dielectric is removed, the two caps are identical and will store the same amount of charge (to ensure they each have the same potential difference).
Total charge is unaffected since the dielectric is removed while the switch is open.
Some charge from C_2 must migrate to C_1 to balance the potential differences while keeping total charge unaffected.
Charge on C_1 increases! Charge on C_2 decreases!

4c) Charge increases on C_1 while its capacitance stays the same.
This implies ΔV_1 must increase!
In turn, because C_2 is in parallel with C_1, we know ΔV_2 also increases.
Remember, when the switch is open ΔV can change!
In this case a human being does work to remove the dielectric causing the energy change (and change in ΔV).
Another way to think of it:

- C_{eq} drops when removing the dielectric
- Q_{eq} is unchanged while dielectric removed with switch open (no path for charges to leave C_{eq})
- since $\Delta V_{eq} = \frac{Q_{eq}}{C_{eq}}$, we see ΔV_{eq} must increase

***4d) We don't know battery EMF ($\mathcal{E}$) or capacitances C_1 & C_2.
We do know $C_2 = \kappa C_1$ where κ is known.
Work the problem out as if we know these unknown quantities to get a feel for the problem.

Before Dielectric Removed	**After** Dielectric removed (switch open)
$C_{eq} = C(1+\kappa)$ $Q_{eq} = \mathcal{E}C_{eq} = \mathcal{E}C(1+\kappa)$ $U_{eq} = \frac{Q_{eq}^2}{2C_{eq}} = \frac{\mathcal{E}^2 C}{2}(1+\kappa)$	$C'_{eq} = 2C$ $Q'_{eq} = Q_{eq} = \mathcal{E}C(1+\kappa)$ $U'_{eq} = \frac{Q'^2_{eq}}{2C'_{eq}} = \frac{\mathcal{E}^2 C}{4}(1+\kappa)^2$

Now use

$$\%\Delta U = \frac{U_f - U_i}{U_i} \times 100\%$$

$$\%\Delta U = \frac{\frac{\mathcal{E}^2 C}{4}(1+\kappa)^2 - \frac{\mathcal{E}^2 C}{2}(1+\kappa)}{\frac{\mathcal{E}^2 C}{2}(1+\kappa)} \times 100\%$$

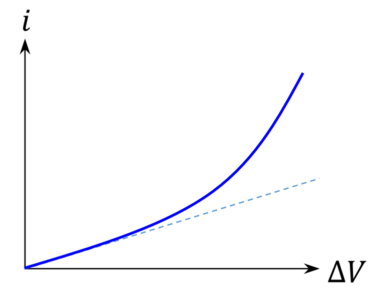
$$\%\Delta U = \left[\frac{1}{2}(1+\kappa) - 1\right] \times 100\%$$

$$\%\boldsymbol{\Delta U} = \mathbf{37.5}\%$$

**5) Carbon is a semi-conductor.
Notice the negative resistivity temperature coefficient on the equation sheet.
As temperature increases, the resistivity of carbon will decrease!
As one applies more voltage, more current flows, more heating occurs.
As one applies more voltage, resistance decreases.
We expect the slope of curve should increase.
Note: strictly speaking, the resistance at any particular voltage is given by

$$R = \frac{\Delta V}{i}$$

Use any point on the curve to compute this value.
For an ohmic device, R is constant and you would get the dashed line shown in the figure.
The slope of that dashed line would be the INVERSE of resistance (called *conductance*).
Notice points above the dashed line have greater conductance (less resistance).



***6) In both scenarios, we see i_1 is the same as battery current.

With the switch *open* we know $R_{eq} = 3R$ (resistors in series).
From there we see $i_1 = i_{eq} = \frac{\mathcal{E}}{3R}$.
The *initial* power delivered to R_1 is thus $\mathcal{P}_1 = i_1^2 R_1 = \frac{\mathcal{E}^2}{9R}$.

With the switch *closed*, we know $R_{34} = \frac{R}{2}$ (in parallel) which gives $R'_{eq} = \frac{5}{2}R = 2.5R$.
From there we see $i_1' = i_{eq}' = \frac{2\mathcal{E}}{5R}$.
The *final* power delivered to R_1 is thus $\mathcal{P}_1' = {i_1'}^2 R_1 = \frac{4\mathcal{E}^2}{25R}$.

Taking a ratio gives the factor:

$$f = \frac{\mathcal{P}_1'}{\mathcal{P}_1}$$

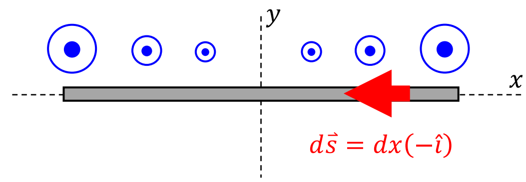
$$f = \frac{\frac{4\mathcal{E}^2}{25R}}{\frac{\mathcal{E}^2}{9R}}$$

$$f = \frac{36}{25}$$

$$\boldsymbol{f = 1.440}$$

7a)

$$[\alpha] = \frac{[B]}{[x^2]} = \frac{\mathrm{T}}{\mathrm{m}^2}$$



7bc) The field gets stronger as you move left or right from the origin.
The magnetic field points out of the page (see figure at right).

$$\vec{F} = \int_{-\frac{L}{2}}^{+\frac{L}{2}} I\, d\vec{s} \times \vec{B}_{ext}$$

$$\vec{F} = \int_{-\frac{L}{2}}^{+\frac{L}{2}} I\, dx(-\hat{\imath}) \times \left(\alpha x^2 \hat{k}\right)$$

$$\vec{F} = I\alpha\left(-\hat{\imath} \times \hat{k}\right) \int_{-\frac{L}{2}}^{+\frac{L}{2}} x^2\, dx$$

$$\vec{F} = 2I\alpha(+\hat{\jmath}) \int_{0}^{\frac{L}{2}} x^2\, dx$$

$$\vec{F} = 2I\alpha(+\hat{\jmath}) \left[\frac{x^3}{3}\right]_0^{\frac{L}{2}}$$

$$\boldsymbol{\vec{F} = \frac{\alpha I L^3}{12}\hat{\jmath}}$$

Check the units…this is why you should always think about the units of your constants.

8a) Rearranging the resistance versus length equation gives

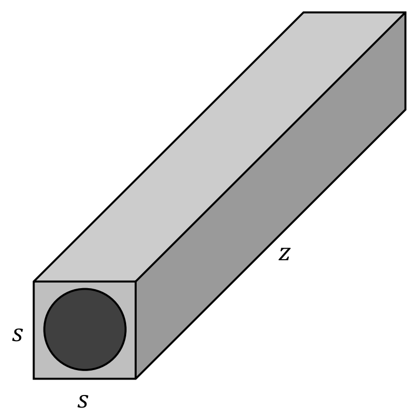
$$R = \frac{\rho L}{A} \quad \rightarrow \quad A = \frac{\rho L}{R}$$

Cross-sectional area is

$$A = s^2 - \frac{\pi}{4} d^2$$

From there one finds

$$d = \sqrt{\frac{4}{\pi}\left(s^2 - \frac{\rho L}{R}\right)}$$



Resistivity of some common metals was given on the equation sheet.
Watch out for all those prefixes!

$$\boldsymbol{d = 8.67\ \mathrm{mm}}$$

8b) Metals have a positive temperature coefficient of resistivity.
As temperature increases, we expect resistance to increase.
Max resistance will occur just before the material melts and loses its shape.

$$R(T) = R_0(1 + \alpha \Delta T)$$

$$R_{660℃} = 333\ \mu\Omega \left(1 + \left(4.4 \times 10^{-3} \frac{1}{℃}\right)(660℃ - 20.0℃)\right)$$

$$\boldsymbol{R_{660℃} = 1271\ \mu\Omega = 1.271\ \mathrm{m}\Omega}$$

Note: on occasion you may notice people write numbers with first digit 1 using a smaller-than-standard prefix.
This is not standard engineering notation but it is not unheard of either…
Usually it is done in the context of a graph (when all the other numbers fit nicely into the smaller prefix).

9a) **Charging occurs more rapidly.** We know because

$$\tau_{charging} = RC < \tau_{discharging} = 2RC$$

9b) Charge is increasing to its max value which occurs while the switch is in position **A**.

9c) Max charge relates to battery voltage using

$$Q = \Delta V C \quad \rightarrow \quad \Delta V = \frac{Q}{C}$$

The plot shows max charge asymptotically approaching 15.00 μC.

$$\Delta V = \frac{15.00\ \mu\text{C}}{567\ \text{nF}} = \frac{15.00 \times 10^{-6}\text{C}}{567 \times 10^{-9}\text{F}} \approx \mathbf{26.5\ V}$$

9c) Using $t = 0_-, t = 0_+, \&\ t = \infty$ pictures (or separation of variables) one determines

$$Q(t) = Q = Q_{max}\left(1 - e^{-t/\tau}\right)$$

I know resistance appears in this equation because $\tau = RC$.
To reduce clutter, I'll solve for τ…then plug in $\tau = RC$ and solve for R.

$$\frac{Q}{Q_{max}} = 1 - e^{-t/\tau}$$

$$e^{-t/\tau} = 1 - \frac{Q}{Q_{max}}$$

$$-\frac{t}{\tau} = \ln\left(1 - \frac{Q}{Q_{max}}\right)$$

$$\tau = -\frac{t}{\ln\left(1 - \frac{Q}{Q_{max}}\right)}$$

We look for a data point on the top approximately midway between the min and max values of charge.
Note: the $\ln\left(1 - \frac{Q}{Q_{max}}\right)$ term will turn out negative and cancel the minus sign up top…

SIDE NOTE: Why is the choice of data point important?
If you pick a value of t close to zero, your result is overly sensitive to the % error associated with reading t.
If you pick a value of Q close to Q_{max}, the result is overly sensitive to the % error associated with reading Q.
Think: what happens to $\ln\left(1 - \frac{Q}{Q_{max}}\right)$ if $Q \approx Q_{max}$…think what happens if you misread the plot slightly???
If you really care about quantifying the errors, take derivatives of τ with respect to Q or ask me to discuss in person.

I noticed charge $Q = 7.00$ μC at time $t = 15$ μs.

$$R = -\frac{t}{C \ln\left(1 - \frac{Q}{Q_{max}}\right)}$$

$$R = -\frac{(15 \times 10^{-6}\text{s})}{(567 \times 10^{-9}\text{F}) \ln\left(1 - \frac{7.00\ \mu\text{C}}{15.00\ \mu\text{C}}\right)}$$

$$\boldsymbol{R \approx 42.1\ \Omega}$$