

**AFTER I GIVE THE SIGNAL TO BEGIN YOU CAN REMOVE THIS SHEET. DO NOT TURN IT IN!**

**163fa21t3a** – Once the exam has officially started, remove the top sheet. The remaining sheets comprise your exam. It is each student's individual responsibility to ensure the instructor has received her or his completed exam. Any exams not received by the instructor earn zero points. Smart watches, phones, or other devices (except scientific calculators) are not permitted during the exam.

$$e = 1.602 \times 10^{-19} \text{ C} \quad k = 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \quad c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \quad \epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$$

$$h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s} \quad hc \approx 1240 \text{ eV}\cdot\text{nm} \quad \mu_0 = 4\pi \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}} \quad 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$m_p = 1.673 \times 10^{-27} \text{ kg} \quad m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$\vec{F} = q\vec{E} \quad k = \frac{1}{4\pi\epsilon_0} \quad \Delta x = v_{ix}t + \frac{1}{2}a_x t^2 \quad v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$

$$\vec{F}_{1on2} = \frac{kq_1q_2}{r_{12}^2} \hat{r}_{1to2} \quad \vec{E} = \frac{kq}{r^2} \hat{r} \quad V = \frac{kq}{r} \quad U_{12} = \frac{kq_1q_2}{r_{12}}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \quad q_{enc} = \int \rho dV \quad E_{\parallel \text{plates}} = \frac{|\Delta V|}{d} = \frac{\sigma}{\epsilon_0} \quad E_{\text{plate}} = \frac{\sigma}{2\epsilon_0}$$

$$E_{ring} = \frac{kQz}{(R^2+z^2)^{3/2}} \quad V_{ring} = \frac{kQ}{(R^2+z^2)^{1/2}} \quad E_x = -\frac{dV}{dx} \quad V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$\Delta U = q\Delta V \quad U_C = \frac{1}{2}Q_C\Delta V_C \quad Q_C = \Delta V_C C \quad I_C = -C \frac{dV_C}{dt}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots \quad C_{eq} = C_1 + C_2 + \dots \quad C_{\text{plates}} = \frac{\epsilon_0 A}{d} \quad C' = \kappa C$$

$$R_{eq} = R_1 + R_2 + \dots \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \quad R = \frac{\rho L}{A} \quad \rho = \rho_0(1 + \alpha\Delta T)$$

$$\Delta V_R = I_R R \quad \mathcal{P}_R = I_R \Delta V_R \quad X(t) = X_f + (X_i - X_f)e^{-t/\tau} \text{ where } \tau = RC \text{ or } \frac{L}{R}$$

$$\vec{F} = q\vec{v} \times \vec{B}_{ext} \quad \vec{F} = I \int d\vec{s} \times \vec{B}_{ext} \quad \vec{\tau} = \vec{\mu} \times \vec{B}_{ext} \quad \vec{\mu} = NI\vec{A}$$

$$U = -\vec{\mu} \cdot \vec{B}_{ext} \quad B_{sol} = \frac{\mu_0 NI}{L} \quad B_{ring} = \frac{\mu_0 I r^2}{2(r^2+z^2)^{3/2}} \quad B_{straight} = \frac{\mu_0 I}{2\pi a}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} \quad I_{enc} = \int \vec{j} \cdot d\vec{A} \quad \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2} \quad \Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$EMF = -N \frac{d}{dt} \Phi_B \quad L = \frac{\Phi_B}{I} \quad U_L = \frac{1}{2} LI^2 \quad \tan \phi = \frac{X_L - X_C}{R}$$

$$\frac{\Delta V_2}{\Delta V_1} = \frac{N_2}{N_1} \quad \Delta V_L = -L \frac{dI_L}{dt} \quad Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{tan } \phi = \frac{X_L - X_C}{R}$$

$$X_L = \omega L \quad X_C = \frac{1}{\omega C} \quad V_{source} = V_0 \sin \omega t \quad i = i_{max} \sin(\omega t - \phi)$$

$$\Delta V_{Rmax} = i_{max} R \quad \Delta V_{Lmax} = i_{max} X_L \quad \Delta V_{Cmax} = i_{max} X_C \quad V_{source \text{ max}} = i_{max} Z$$

$$\Delta V_{max} = \frac{\Delta V_{pk} - pk}{2} \quad \Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \omega = 2\pi f = \frac{2\pi}{T}$$

$$\mathcal{P}_{avg} = I_{rms} \Delta V_{rms} \cos \phi = I_{rms}^2 R \quad I_{avg} = S_{avg} = \frac{E_{max} B_{max}}{2\mu_0} = \left(\frac{1}{c}\right) \frac{E_{max}^2}{2\mu_0} = c \frac{B_{max}^2}{2\mu_0}$$

$$c = f\lambda \quad E_{\gamma} = hf = \frac{hc}{\lambda} \quad \text{Photon momentum} = p_{\gamma} = \frac{E_{\gamma}}{c}$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \quad \frac{E_{max}}{B_{max}} = c \quad \text{Rad. Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{S_{avg}}{c}$$

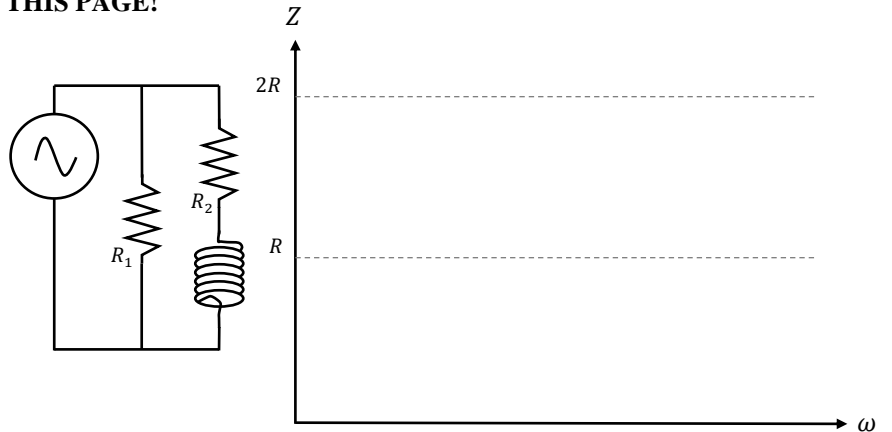
Material	Resistivity at 20° C (in SI units)	Temp. Coefficient (in SI units)
Silver	$1.62 \times 10^{-8}$	$4.1 \times 10^{-3}$
Copper	$1.69 \times 10^{-8}$	$4.3 \times 10^{-3}$
Aluminum	$2.75 \times 10^{-8}$	$4.4 \times 10^{-3}$
Nichrome	$1.00 \times 10^{-6}$	$0.4 \times 10^{-3}$
Carbon	$3.5 \times 10^{-5}$	$-0.5 \times 10^{-3}$
Germanium	0.46	$-48 \times 10^{-3}$

$\int \frac{x dx}{(x^2 + a^2)^{3/2}} = \frac{-1}{\sqrt{x^2 + a^2}}$		$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} $	
$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2 + a^2}} = \frac{1}{a^2} \sin \theta$		$\int \frac{x dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$	
$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$		$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln x + \sqrt{x^2 \pm a^2} $	
$\int \frac{x dx}{x^2 + a^2} = \frac{1}{2} \ln x^2 + a^2 $		<b>Binomial expansion:</b> $(1 \pm \delta)^n \approx 1 \pm n\delta + \dots$	
T = 10 <sup>12</sup>	G = 10 <sup>9</sup>	M = 10 <sup>6</sup>	k = 10 <sup>3</sup>
c = 10 <sup>-2</sup>	m = 10 <sup>-3</sup>	μ = 10 <sup>-6</sup>	n = 10 <sup>-9</sup>
p = 10 <sup>-12</sup>	f = 10 <sup>-15</sup>	a = 10 <sup>-18</sup>	

**WRITE YOUR NAME AT THE TOP OF THIS PAGE!**

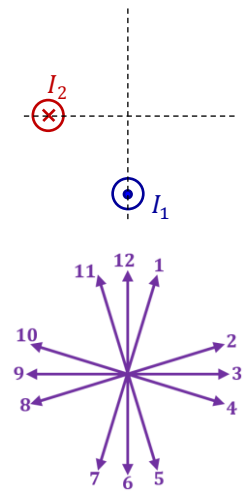
A circuit contains an ideal AC source operating with variable frequency  $\omega$ . The circuit uses two identical resistors with resistance  $2R$  and an inductor with inductance  $L$ . The resistors are labeled  $R_1$  and  $R_2$  for ease of communication.

\*\*1) Sketch a plausible curve representing total impedance as a function of  $\omega$  using the plot at right.



Two wires carry currents  $I_1 = I$  &  $I_2 = 2I$  in the directions shown in the *upper* figure at left. To be clear, you may assume the wires run into and out of the page infinitely far in both directions. The wires are equidistant from the origin.

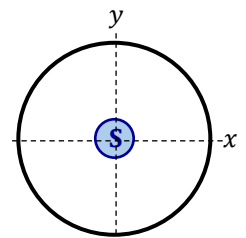
2) Which of the numbered arrows (shown in the *lower* figure at left) best represents the direction of the total magnetic field at the origin?



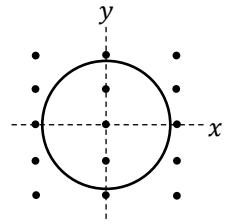
A magnetic bullet moves away from an aluminum loop of wire with constant velocity  $-v\hat{k}$ . In the figure at right, the south pole of the bullet is just behind the plane of the loop moving into the page. The loop of wire contains has no battery or power source.

3) As the bullet moves away from the loop of wire, would we expect any current or no? If current does flow, which direction is current flow? Circle the best answer.

No current	Current flows CW	Current flows CCW	Impossible to determine without more information
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An aluminum wire 5.00 m long is used to make a 75.0 turn circular coil. The wire's cross-sectional diameter is 0.125 mm. The circular coil is centered on the origin as shown in the figure. A uniform external field with magnitude 333 mT is directed out of the page as shown. Starting from rest at time  $t = 0$ , the loop rotates with constant angular *acceleration*  $22.2 \frac{\text{rad}}{\text{s}^2}$  about the  $y$ -axis.



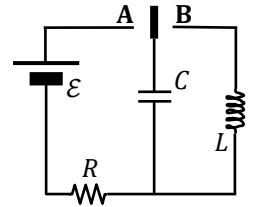
4a) Determine the radius of the circular coil shown in the figure.

\*\*\*4b) Determine an induced emf in the loop at  $t = 4.44$  s.

For partial credit, derive an algebraic expression.

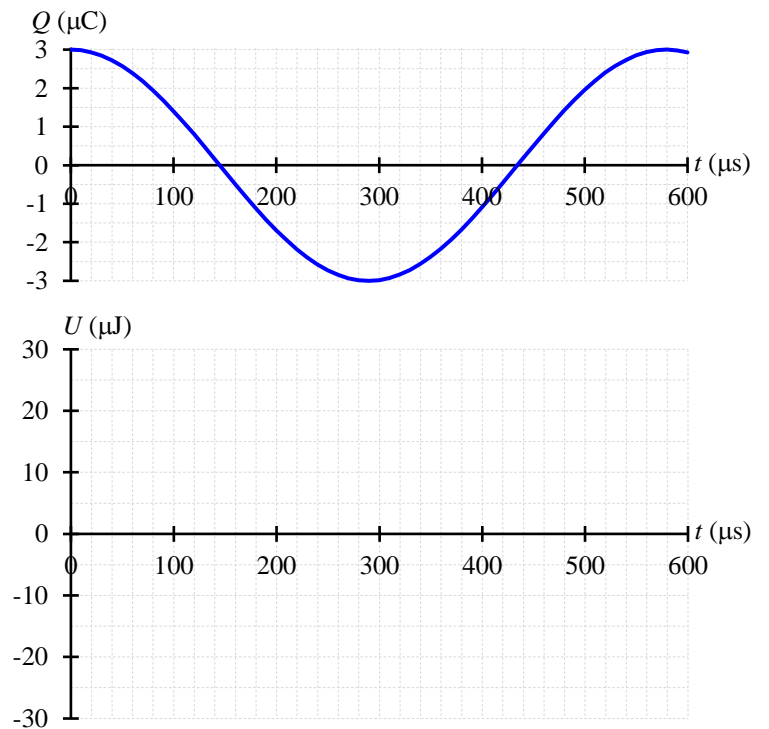
4a	
4b	

A circuit is built using capacitance  $C = 250.0 \text{ nF}$  as shown in the schematic shown at right. The capacitor is allowed to charge in position **A** for a long time. At time  $t = 0$ , the switch is thrown to position **B**. A plot of capacitor charge as a function of time is shown near the bottom of the page.

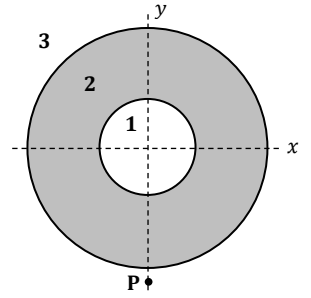


- 5a) Determine the battery voltage  $\mathcal{E}$  used to initially charge the capacitor.
- 5b) At what time does current through the inductor first reach a maximum?
- \*\*5c) Determine the inductance  $L$ .
- \*\*5d) Use the empty plot below to sketch potential energy in the capacitor versus time.

5a	
5b	
5c	



A cylindrical shell carries non-uniform current density  $J = kr^3$  where  $k$  is an unknown positive constant. A cross section of the shell is shown at right (figure not to scale). The shell has outer radius  $R$  and inner radius  $R/2$ . Total current in the shell is  $I$ . The three obvious regions are labeled with numbers in the figure to ease communication.  $\mathbf{P}$  is located beneath the shell as shown in the figure.

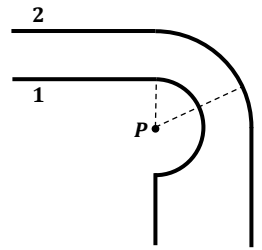


- 6a) Determine the units appropriate for the constant  $k$ .
- 6b) Determine  $k$ . Express your result as a decimal number with 3 sig figs times an algebraic expression using only known quantities.
- 6c) Which direction does current flow if the magnetic field direction at  $\mathbf{P}$  points to the left?
- 6d) Determine magnetic field magnitude for an arbitrary radius  $r$  in region **1**.
- 6e) Determine magnetic field magnitude for an arbitrary radius  $r$  in region **2**.
- 6f) Determine magnetic field magnitude for an arbitrary radius  $r$  in region **3**.

6a	
6b	
6c	
6d	
6e	
6f	

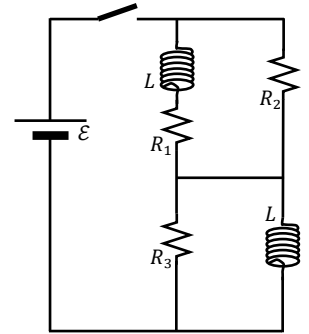
Two wires are bent into the shapes shown in the figure (not to scale). Current in wire 2 is twice as large as current in wire 1. To be clear, you may assume the all straight line segments extend to infinity. Notice point **P** lies at the center of the circular segments of each wire. The arrangement is designed to ensure the net magnetic field at **P** is zero.

\*\*\*7) Determine the ratio of the larger radius to the smaller radius. Express your answer as a number with three sig figs. If such a scenario is impossible, do not use the answer box. Instead, explain why the scenario is impossible and support your conclusion with calculations.



7	
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A circuit is built as shown in the schematic at right. Both inductors have inductance  $L$  and the battery is ideal with potential difference  $\mathcal{E}$ . At time  $t = 0$  the switch is closed and the circuit is allowed to reach steady state. The resistors are numbered for ease of communication. All resistors have resistance  $R$ .



\*\*8a) Determine the current through resistor  $R_3$  just after the switch is closed.

\*\*8b) Determine the power delivered by the battery after the circuit reaches steady state.

8a	
8b	

A series *LRC* circuit employs a function generator operating at 1.880 kHz with voltage amplitude 7.77 V. The circuit uses 66.6  $\Omega$  of resistance and 555 nF of capacitance. The average power delivered to the resistor at this operating frequency is 444 mW.

\*\*9a) Determine current amplitude at this operating frequency.

\*\*\*9b) Determine the inductance used in this circuit.

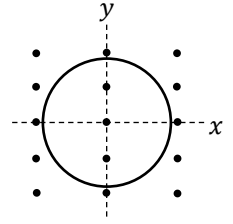
9a	
9b	



**\*\*EXTRA CREDIT:**

Reconsider the problem which had a loop rotating in an external magnetic field with constant angular acceleration. In particular, consider the phase angle between current and the induced EMF for this scenario.

1. Should the phase angle be positive, negative, or zero? Explain why.
2. Will the phase angle change over time or not? If it doesn't change, explain why not. If it does change, state if it becomes more positive or more negative AND explain why?



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