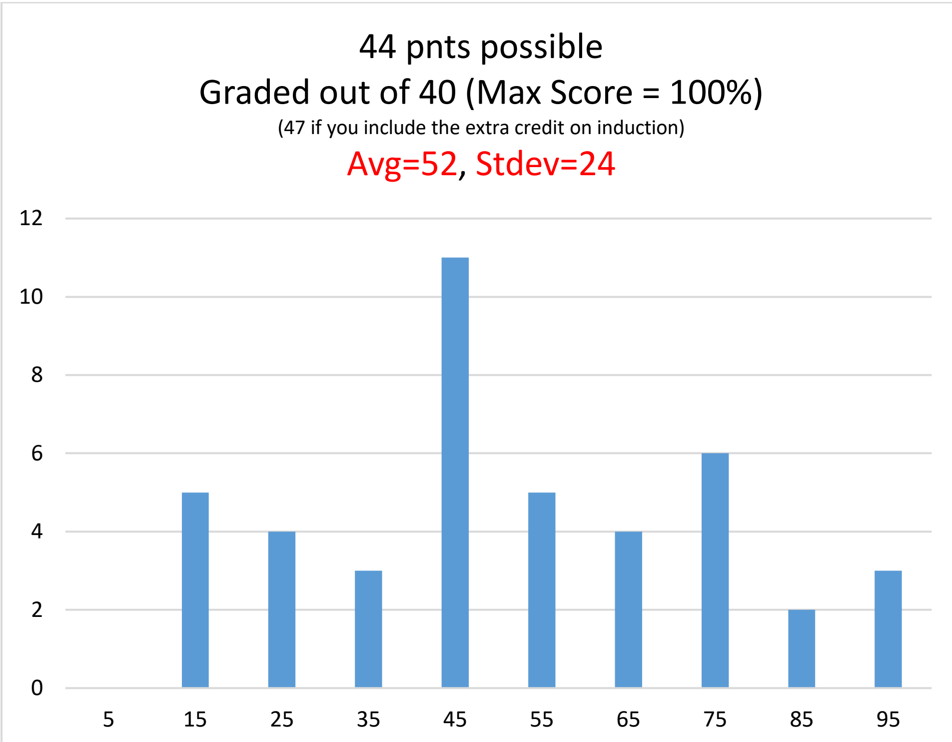
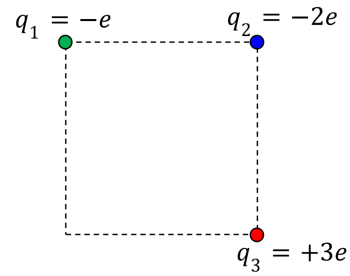
163fa22t1aSoln

Distribution on this page, solutions begin on next page.
I don't write up answers to extra credit questions.



1a) Side length of square is s. Main diagonal length is $\sqrt{2}s$.



$$U_{total} = U_{12} + U_{13} + U_{23}$$

$$U_{total} = \frac{k(-e)(-2e)}{s} + \frac{k(-e)(3e)}{\sqrt{2}s} + \frac{k(-2e)(3e)}{s}$$

$$U_{total} = -6.12\frac{ke^2}{s}$$

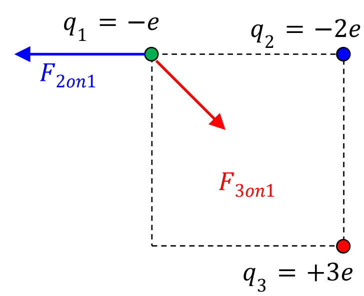
1b) Side length of square is s. Main diagonal length is $\sqrt{2}s$.

$$V_{total} = V_1 + V_2 + V_3$$

$$V_{total} = \frac{k(-e)}{s} + \frac{k(-2e)}{\sqrt{2}s} + \frac{k(3e)}{s}$$

$$V_{total} = 0.586\frac{ke}{s}$$

1c) See the figures at right. **Final answer in the figure.**

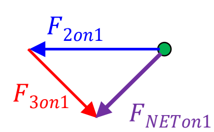


$$\vec{F}_{2on1} = \frac{k(-e)(-2e)}{s^2}(-\hat{\imath}) = -\frac{2ke^2}{s^2}\hat{\imath}$$

$$\vec{r}_{3to1} = -s\hat{\imath} + s\hat{\jmath}$$

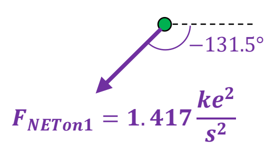
$$r_{3to1} = \sqrt{2}s$$

$$\vec{F}_{3on1} = \frac{k(-e)(3e)}{\left(\sqrt{2}s\right)^3}(-s\hat{\imath} + s\hat{\jmath})$$



$$\vec{F}_{3on1} = \frac{3\sqrt{2}}{4}\cdot\frac{ke^2}{s^2}(\hat{\imath} - \hat{\jmath})$$

Side note: **WATCH OUT!**



$$F_{3on1} \neq \frac{3\sqrt{2}}{4}\cdot\frac{ke^2}{s^2}$$

The x & y components are of equal size; the *magnitude* is

$$F_{3on1} = \sqrt{2}\left(\frac{3\sqrt{2}}{4}\cdot\frac{ke^2}{s^2}\right) = 1.500\frac{ke^2}{s^2}$$

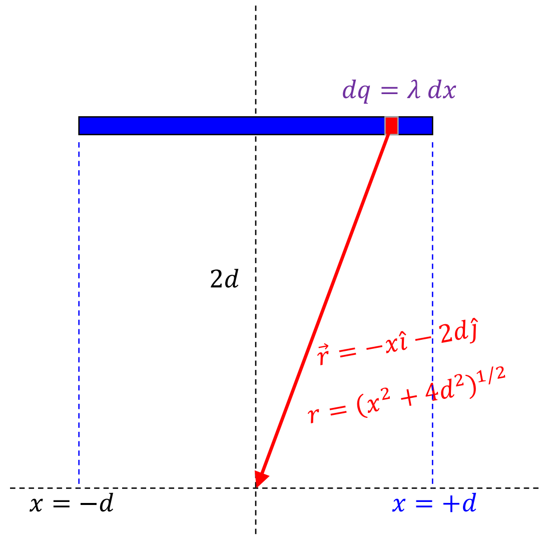
Now add the force *vectors*…then get the *magnitude* of the net.

$$\vec{F}_{NETon1} = \vec{F}_{2on1} + \vec{F}_{3on1} = -\frac{2ke^2}{s^2}\hat{\imath} + \left(\frac{3\sqrt{2}}{4}\cdot\frac{ke^2}{s^2}\hat{\imath} - \frac{3\sqrt{2}}{4}\cdot\frac{ke^2}{s^2}\hat{\jmath}\right)$$

$$\vec{F}_{NETon1} = (-0.9393\hat{\imath} - 1.0607\hat{\jmath})\frac{ke^2}{s^2} \quad\rightarrow\quad F_{NETon1} = 1.417\frac{ke^2}{s^2}$$

The problem statement asked for an angle "between $-180°$ & $+180°$ from the standard positive x-axis"**.**

2) Charge density is



$$\lambda = \frac{Q}{2d}$$

$$\vec{E} = \int_{-d}^{d} d\vec{E}_{point\ charge}$$

$$\vec{E} = \int_{-d}^{d} \frac{k\, dq\, \vec{r}}{r^3}$$

$$\vec{E} = \int_{-d}^{d} \frac{k\left(\frac{Q}{2d}\, dx\right)(-x\hat{\imath} - 2d\hat{\jmath})}{(x^2 + 4d^2)^{3/2}}$$

At the origin, we expect the horizontal component of the field to cancel.

$$\vec{E} = (-2d\hat{\jmath})k\left(\frac{Q}{2d}\right)\int_{-d}^{d} \frac{dx}{(x^2 + 4d^2)^{3/2}}$$

$$\vec{E} = (-\hat{\jmath})kQ\int_{-d}^{d} \frac{dx}{(x^2 + 4d^2)^{3/2}}$$

Notice we can use the following integral from the equation sheet if we identify $a = 2d$:

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}}$$

$$\vec{E} = kQ\left[\frac{x}{(2d)^2\sqrt{x^2 + (2d)^2}}\right]_{-d}^{d} (-\hat{\jmath})$$

Optional: To further simplify the integral, one could identify it as an even function with symmetric limits.

$$\vec{E} = \mathbf{2}kQ\left[\frac{x}{(2d)^2\sqrt{x^2 + (2d)^2}}\right]_{\mathbf{0}}^{d} (-\hat{\jmath})$$

Doing some math, one finds

$$\vec{E} = \mathbf{2}kQ\left(\frac{d}{4d^2\sqrt{d^2 + 4d^2}}\right)(-\hat{\jmath})$$

$$\vec{E} = -\frac{kQ}{2\sqrt{5}d^2}\hat{\jmath}$$

$$\boldsymbol{\vec{E} = -0.224\frac{kQ}{d^2}\hat{\jmath}}$$

3a) & 3b) The given function is

$$E_x = \frac{c}{b^2 + x^2}$$

First consider the denominator.
Whenever two terms are summed, we require them to have identical units.
I think of this as "you can only add apples to apples".
Therefore $[b] = [x] = \mathrm{m}$.
After that is figured out, we know

$$[c] = [E] \cdot [x]^2 = \frac{\mathrm{V}}{\mathrm{m}} \cdot \mathrm{m}^2 = \mathrm{V} \cdot \mathrm{m}$$

Alternate acceptable answers: many of you probably wrote

$$[c] = [E] \cdot [x]^2 = \frac{\mathrm{N}}{\mathrm{C}} \cdot \mathrm{m}^2$$

If you are feeling frisky you could then use $1\ \mathrm{N} = 1 \frac{\mathrm{kg{\cdot}m}}{\mathrm{s}^2}$ but I didn't bother with that (gave full credit if you did).

That said the problem talks about potential and field…seems better to use the units of $\frac{\mathrm{V}}{\mathrm{m}}$ for the field.

3c) Use

$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$$

In this instance:

- $\vec{E} = \frac{c}{b^2+x^2}\hat{\imath}$
- $d\vec{s} = dx\ \hat{\imath}$ (problem implied $dx\ \hat{\imath}$ when it said "between the origin & a point distance d to the right")
- $x_i = 0$ (started at the origin)
- $x_f = d$

$$\Delta V = -\int_0^d \left(\frac{c}{b^2 + x^2}\hat{\imath}\right) \cdot (dx\ \hat{\imath})$$

$$\Delta V = -c \int_0^d \frac{dx}{b^2 + x^2}$$

Use integral below from equation sheet (identify variable a in the table with variable b in the E-field expression…)

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

One finds

$$\Delta V = -c\left[\frac{1}{b}\tan^{-1}\frac{x}{b}\right]_0^d$$

Always check both limits, even when one of them is zero! In this case $\tan^{-1}(0) = 0$.

$$\boldsymbol{\Delta V = -\frac{c}{b}\tan^{-1}\left(\frac{d}{b}\right)}$$

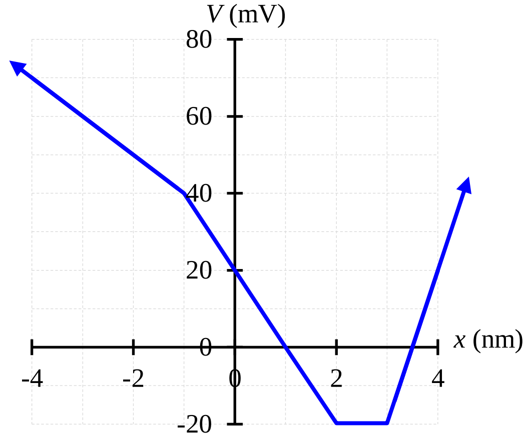
Check the units using the results of parts a & b. Notice argument of function is unitless.

Think: This is tricky one to check the sign on. Field points to the *right* ($E_x > 0$). Need higher voltage on the *left*. Since moving to the *right*, require $\Delta V < 0$.

4a) The statement "strongest field" implies largest electric field magnitude.
This occurs wherever the plot has steepest slope.

Answer: between +3 nm and +4 nm (or between +3 nm and $+\infty$)

If you left off units I probably dinged your score!



4b) The axis of the plot implies motion is only occurring in the x-direction.

$$E_x = -\frac{dV}{dx}$$

$$E_x = -slope$$

$$E_x = -(-\#)$$

$$E_x > 0$$

$$F_x = (-e)E_x$$

$$F_x < 0$$

Force *to the left* on the *electron*.

An electron released from rest at $x = 1.0$ nm moves *to the left*.

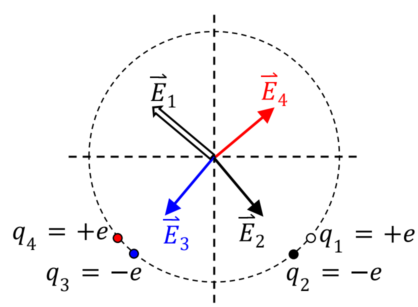
4c) An electron released from rest at $x = 1.0$ nm moves *to the left*.
Notice the slope is less steep after the electron passes $x = -1.0$ nm.
This implies force (and thus acceleration decreases) when the electron passes $x = -1.0$ nm.
The best answer available is: Moves away, *acceleration decreases*, never to return

OPTIONAL: We know if this goes on forever the electron gradually nears the speed of light.
Acceleration must gradually decrease despite constant force.
Found a vid explaining it https://www.youtube.com/watch?v=S10GWreRtug.
Turns out velocity as a function of time for constant force (while accounting for relativity) is given by

$$v(t) = \frac{ct}{\sqrt{\left(\frac{mc}{F}\right)^2 + t^2}}$$

5a) Zero potential (same amount of + and – charges equidistant from origin).

5b) $E_{NETx} = 0$. Notice horizontal components cancel in the figure at right.

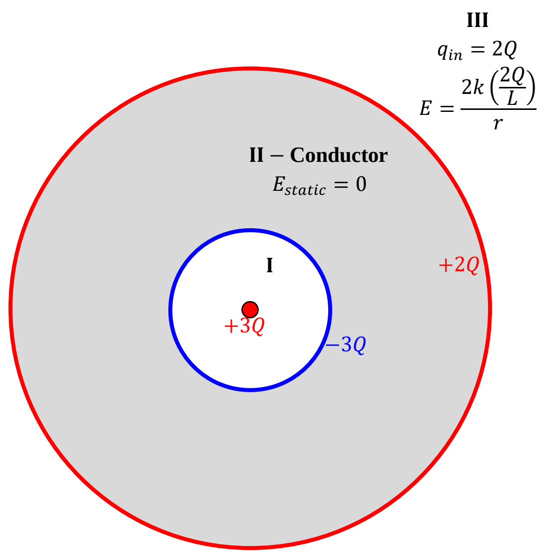


5c) $E_{NETy} < 0$. Notice the downwards components from $\vec{E}_2$ & $\vec{E}_3$ are slightly larger than the upwards components from $\vec{E}_1$ & $\vec{E}_4$.

5d) To get an exact result, we must consider all pairs. However, $U = \frac{kQq}{r}$.
Notice this blows up for small charge separation (small values for r)
Clearly the two terms U_{34} & U_{12} will dominate in any potential energy calculation.
Both of those terms are negative and enormous compared to all other terms.
Going further: notice the terms U_{13} & U_{14} will almost exactly cancel since $r_{13} \approx r_{14}$. Similar for other terms.
Electric potential energy must be *negative* for this assembly of charge.

6a) When handling this type of problem, work from the inner most surface and work your way outwards.

The conductor (**Region II**) will strongly polarize.
The positive line charge at the center will draw $-3Q$ to the inner surface of the conductor.
The net charge on the conductor must be $-Q$ from the problem statement.
All excess charge is on the surfaces of the conductor.
Therefore we require $+2Q$ on the outer surface.
Even though the conductor carries *total* charge $-Q$, neither surface has charge $-Q$!!!



6b) Want surface charge density of cylindrical surface

$$\sigma = \frac{+2Q}{2\pi(\mathbf{3R})L}$$

$$\boldsymbol{\sigma = \frac{Q}{3\pi RL}}$$

6c) Remember: in Gauss's law problems, only charge enclosed determines to the net electric field. By symmetry, the conductor will have no effect! Here we can simply use our memorized result with $\lambda_1 = +\frac{3Q}{L}$.

$$E_I = \frac{2k\lambda_1}{r}$$

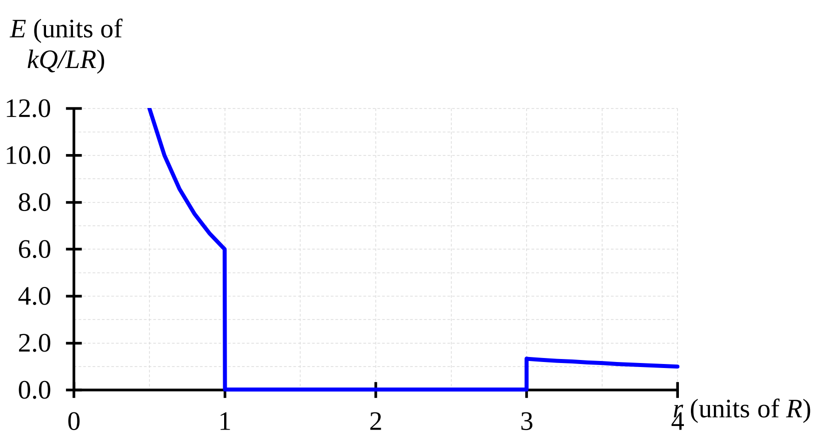
$$\boldsymbol{E_I = \frac{6kQ}{rL}}$$

6d) Zero field (zero *static* field) inside a conductor.

6e) Both regions are fully enclosed! We can again use our memorized result with $\lambda_{total} = \frac{Q_{total}}{L} = \frac{2Q}{L}$.

$$E_{III} = \frac{2k\lambda_{total}}{r}$$

$$\boldsymbol{E_{III} = \frac{4kQ}{rL}}$$



Optional I filled in this table to help me figure out the plot	
Radius	**Field Mag**
$r = \frac{R}{2}$	$E_I = \frac{12kQ}{RL}$
$r = R$ for region I	$E_I = \frac{6kQ}{RL}$
$r = R$ for region II	$E_{II} = 0$
$r = 2R$	$E_{II} = 0$
$r = 3R$ for region II	$E_{II} = 0$
$r = 3R$ for region III	$E_{III} = \frac{4kQ}{3RL}$
$r = 4R$	$E_{III} = \frac{kQ}{RL}$

7) MOST COMMON QUESTION: I didn't specify a position for q2. The computer with thus use the default position <0,0,0>. Don't believe me? Type up the code and do a print statement for q2.pos. It will return <0,0,0>.

```
1  GlowScript 3.1 VPython
2  #assume SI units on all numbers
3
4  q1 = sphere()
5  q1.pos = vec(1.245, 2.346, -3.03)
6  q1.mass = 1e-6
7  q1.velocity = vec(0,0,0)
8  q1.acceleration = vec(0,0,0)
9  q1.charge = 1e-8
10
11 q2 = sphere()
12 q2.mass = 1e-6
13 q2.velocity = vec(0,0,0)
14 q2.acceleration = vec(0,0,0)
15 q2.charge = 1e-8
16
```

We were asked to write code to compute and *output* the electric field *magnitude* at the point $(2.22, -7.77, 15.38)$. One possible way to do this is shown below.

- Did you remember to define `k`?
- If you chose to do something like POI.pos, you must first define POI as some kind of object or the code will not compile!
  - For example, you could do this:
    ```
    POI = sphere()
    POI.pos = vec(2.22, -7.77, 15.38)
    ```
- Did you correctly determine the r-vector for each charge?
- Did you do the following:
  - Get electric field VECTOR for each charge
  - Add the vectors together
  - THEN take the magnitude?
- Did you remember to output the result with a print statement?
- Normally we *should* include a string to help clarify/label the print statement but it was not *required* for the test question.

Line 17	`POI = vec(2.22, -7.77, 15.38)`
Line 18	`k = 8.99e9`
Line 19	`r1 = POI - q1.pos`
Line 20	`r2 = POI - q2.pos`
Line 21	`E1 = k*q1.charge*r1/mag(r1)**3`
Line 22	`E2 = k*q2.charge*r2/mag(r2)**3`
Line 23	`E_NET = E1 +E2`
Line 24	`print(“E_NET = ” + mag(E_NET) + “ N/C”)`

8a) Gravitational force (magnitude) on an electron near earth's surface is $F_g = m_e g \approx 10^{-29}$.
Electrical force (magnitude) on an electron in an electric field is $F_E = eE = (1.602 \times 10^{-19}\ \text{C})E$.
Even extremely weak electric fields (as small as $10\ \frac{\text{nV}}{\text{m}}$) are large enough to dwarf gravitational forces.
For ions, protons, and electrons we typically ignore gravitational forces compared to electrical forces due to their tiny masses.

8b) The electron is deflected to the *left*. Force on the electron must be to the *left*.
For *negative* charges, *force* points *opposite* the direction of the electric *field*.
Therefore, electric *field* points to the *right*.

8c) Electric *field* points to the *right*. Electric fields point from high voltage to low voltage.
Therefore, the *left* plate must be held at higher potential.

8d) In this scenario, the electron experiences a constant force (and thus constant acceleration).
It is actually ok to use kinematics for this problem (although it could also be partially solved with energy methods).
Think: horizontal acceleration does not affect the vertical motion due to independence of xy-motion.
From kinematics in the *vertical* direction we know

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$-100d = (-v_0)t + \frac{1}{2}(0)t^2$$

$$t = \frac{100d}{v_0}$$

To determine the *horizontal acceleration* I did the following:

$$a_x = \frac{F_x}{m} = \frac{qE_x}{m} = \frac{(-e)\left(\frac{-\Delta V}{spacing}\right)}{m} = \frac{e\Delta V}{5md}$$

Note: you may wonder why this acceleration *appears* to be positive (even though it is supposed to be negative).
It is because $\Delta V < 0$ when computing from left to right. Remember, in this problem $V_{left\ plate} > V_{right\ plate} \ldots$
To make it look better, use

$$a_x = -\frac{e|\Delta V|}{5md}$$

Here I am calling $|\Delta V|$ the *magnitude* of the potential difference between the plates.
Using kinematics in the *horizontal* direction gives:

$$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$$

$$-3d = (0)t + \frac{1}{2}\left(-\frac{e|\Delta V|}{5md}\right)t^2$$

Notice the electron's horizontal displacement is not the entire spacing but rather $3d$ to the left…
Solving for ΔV gives

$$|\Delta V| = \frac{30md^2}{et^2}$$

Using the result for time from vertical kinematics one finds.

$$|\Delta V| = \frac{30md^2}{e\left(\frac{100d}{v_0}\right)^2} = 3 \times 10^{-3}\,\frac{mv_0^2}{e}$$

The quick way to check the units: We know $\Delta U = energy\ change = q\Delta V$.
The units of Volts should be energy per unit charge. I recognize mv_0^2 as having the units of kinetic energy…
Units check out ok.