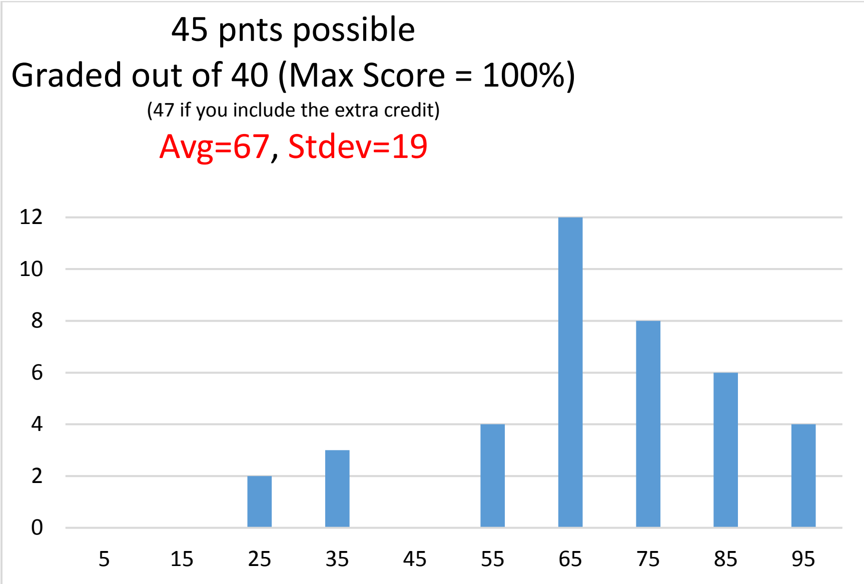
163fa22t2aSoln



Distribution on this page, solutions begin on next page.

1a) METAL

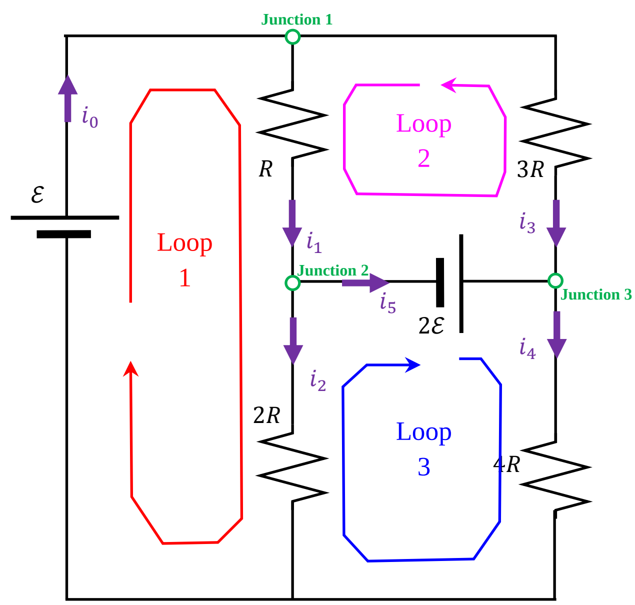
Typically the resistance & resistivity of *metals* will *increase* as they get hotter.
Typically the resistance & resistivity of *semi-conductors* will *decrease* as they get hotter.

1b) Use $R = \frac{\rho L}{s^2}$ because the cross-sectional area is $A = s^2$.
I found $\boldsymbol{s = 365\ \mu m}$.

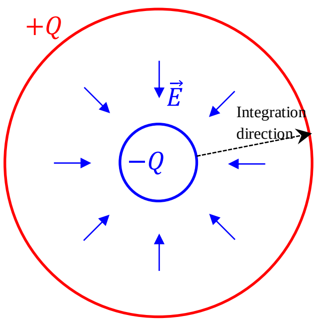
1c) We know $R_{hot} = 1.425 R_0$.
Be careful… $57.25\% R_{hot}$ *IS NOT* R_0 …it is *similar to* but not the same as the equation above.
Also, $R_{hot} = R_0[1 + \alpha(T_C - 20.0℃)]$.
WATCH OUT! In the previous equation R_0 is resistance at $20.0℃$…NOT resistance at $0℃$.
I found $\boldsymbol{\alpha = 7.52 \times 10^{-4} \frac{1}{℃}}$.
Expect to lose points if you forgot units!!!

2) Using the labels at right I found

Loop 1	$\mathcal{E} - i_1 R - 2 i_2 R = 0$
Loop 2	$2\mathcal{E} + 3 i_3 R - i_1 R = 0$
Loop 3	$2\mathcal{E} - 4 i_4 R + 2 i_2 R = 0$
Junction 1	$i_0 = i_1 + i_3$
Junction 2	$i_1 = i_2 + i_5$
Junction 3	$i_3 + i_5 = i_4$



3) Capacitance is essentially the amount of charge a pair of conductors can store *per unit volt*. Therefore we, need an expression that involves both charge and voltage.



First, assume each conductor carries some amount of charge Q.
I assume the *inner* conductor has *negative* charge (to cancel a minus sign later).

Realize that for a given *spherical* charge, we know the electric field (from Gauss's law).

$$\vec{E} = -\frac{kQ\hat{r}}{r^2}$$

Think: as you move away from the central conductor, the field must get weaker…it must involve the variable distance r (not the constant radius R of the inner shell).

Get potential difference between the conductors using $\Delta V = -\int_i^f \vec{E}\cdot d\vec{s}$.

$$\Delta V = -\int_R^{4R}\left(-\frac{kQ\hat{r}}{r^2}\right)\cdot dr\ \hat{r}$$

$$\Delta V = \left[-\frac{kQ}{r}\right]_R^{4R}$$

$$\Delta V = \frac{3kQ}{4R}$$

From this one determines

$$\boldsymbol{C = \frac{Q}{\Delta V} = \frac{4R}{3k} = \frac{16}{3}\pi\epsilon_0 R}$$

4a) The time constant $\tau = R_{eq}C_{eq}$ gives the time scale in these types of questions.
In both switch positions, the circuit has equivalent resistance $R_{eq} = 2R$.
Both charging and discharging should occur at the same rate.

4b) As the capacitor *charges* (in position **A**) we expect current to initially be large then gradually fall off.
As the capacitor dis*charges* (in position **B**) we ALSO expect current to initially be large then gradually fall off.
We cannot tell if the plot was made during charging or discharging…
Either position could produce such a plot.

4c) Let us assume charging since either switch position is valid.
I will assume the capacitor is initially uncharged at time $t = 0_-$ (just *before* switch flipped to position **A**).
A capacitor preserves its voltage during switching.
Therefore, the capacitor has no potential difference at time $t = 0_+$ (just *after* switch flipped to position **A**).
By KVL, the sum of all voltage gains around the loop must be zero.
This implies the voltage drop across the resistor is initially the same as the battery.
After a long time in position **A** (at time $t = \infty$), the capacitor is fully charged.
A fully charged capacitor blocks current flow (acts like a break in the circuit).
With no current flow, resistor voltage is zero!
Use

$$\Delta V_R = \Delta V_{R\,f} + \left(\Delta V_{R\,i} - \Delta V_{R\,f}\right)e^{-t/\tau}$$
$$\Delta V_R = \mathcal{E}e^{-t/\tau}$$

In this expression, $\Delta V_{Ri} = \frac{\mathcal{E}}{2}$ is the *half* of the battery's potential difference.
Notice, from the plot, one finds $\Delta V_{R\,i} = \frac{\mathcal{E}}{2} = 750$ mV.

The standard technique at this point is to look for a data point in the sweet spot of the graph (when the voltage has dropped by approximately 50%...but also when the data lines up close to a gridline).
I noticed that the plot reads almost exactly 250 mV at 8.0 ns…these numbers are easily good to 2 sig figs which should give us errors of 10% or less. Another good spot is probably 500 mV at 3.0 ns.
Tip: I like to plug in voltages immediately but plug in the time at the very end (after taking natural log).

$$250\text{ mV} = (750\text{ mV})\,e^{-t/\tau}$$
$$0.33\underline{3}3 = e^{-t/\tau}$$
$$\ln 0.33\underline{3}3 = -\frac{t}{\tau}$$
$$\tau = -\frac{t}{\ln 0.33\underline{3}3}$$
$$(2R)C = -\frac{t}{\ln 0.33\underline{3}3}$$
$$R = -\frac{t}{2C\ln 0.33\underline{3}3}$$
$$R = -\frac{8.0\times10^{-9}\text{ s}}{2(4.70\times10^{-12}\text{ F})\ln 0.33\underline{3}3}$$
$$\boldsymbol{R = 77\underline{4}.6\ \Omega}$$

4d) Initial power delivered to the resistor on the central branch is

$$\mathcal{P}_i = \Delta V_{R\,i}\,i = \frac{(\Delta V_{R\,i})^2}{R} = \frac{(750\text{ mV})^2}{77\underline{4}.6\ \Omega} \approx \mathbf{726\ \mu W}$$

5a) We know resistor in parallel can be combined using

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{N3}}$$

For N *identical* resistors of resistance r in parallel the equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{r} + \frac{1}{r} + \cdots + \frac{1}{r} = \frac{N}{r}$$

Flipping the fraction gives

$$R_{eq} = \frac{r}{N}$$

Note, in this particular set of circuits we must remember to add the additional resistance R in series to get $R_{eq\ circuit}$.

Initially, with the switch *closed*, we know $R_{eq\ circuit} = R + \frac{r}{4}$ which gives total power delivered by the battery

$$\mathcal{P}_i = \frac{\mathcal{E}^2}{R_{eq\ circuit}} = \frac{\mathcal{E}^2}{R + \frac{r}{4}}$$

Afterwards, with the switch *open*, we know $R_{eq} = R + \frac{r}{3}$ which gives total power delivered by the battery

$$\mathcal{P}_f = \frac{\mathcal{E}^2}{R_{eq\ circuit}} = \frac{\mathcal{E}^2}{R + \frac{r}{3}}$$

Finally, we were given $\mathcal{P}_f = f\mathcal{P}_i$ where the factor is $f = 0.925$.

$$\mathcal{P}_f = f\mathcal{P}_i$$

$$\frac{\mathcal{E}^2}{R + \frac{r}{3}} = f\frac{\mathcal{E}^2}{R + \frac{r}{4}}$$

$$R + \frac{r}{4} = f\left(R + \frac{r}{3}\right)$$

$$R + \frac{r}{4} = fR + \frac{f}{3}r$$

$$R - fR = \frac{f}{3}r - \frac{r}{4}$$

$$R(1-f) = r\left(\frac{f}{3} - \frac{1}{4}\right)$$

$$r = R\frac{(1-f)}{\left(\frac{f}{3} - \frac{1}{4}\right)}$$

$$\boldsymbol{r = 1.286R}$$

5b) When the switch is *opened*, a path for current to travel is *removed* from the circuit.
This implies more equivalent resistance and less total current (**since battery voltage remains constant**).
Since battery power is proportional to current times voltage (and voltage remains unchanged), power must also drop (factor less than 1).

Many students would say things like “Resistance went up so power must go down” or “Power is inversely proportional to resistance” but did not add the crucial clarification about voltage remaining constant. Some students would talk about removing resistance then say current goes down. By itself this is very confusing as well. I looked for clear communication…not just regurgitating memorized phrases without adequate context.

6a) Once steady state is reached, both capacitors are charged.
When charged in this manner (wired in parallel), we expect both caps to have the same potential difference.
We were asked about energy and know both caps have the same voltage…makes sense to rewrite the energy equation in terms of voltage to simplify the comparison!

$$U = \frac{1}{2}Q\Delta V = \frac{\Delta V^2 C}{2}$$

The *larger* cap should have more energy *when they have the same potential difference*.

6b)

$$C_{eq} = \frac{10}{7}C \approx 1.429C$$

6c) Once we know C_{eq}, we can find charge on the equivalent capacitor is $Q_{eq} = \frac{10}{7}\mathcal{E}C$.
I don't know the battery voltage $\mathcal{E}$ yet…but I do know the charge on the equivalent circuit happens to be the charge on $2C$ in this scenario (since C_{eq} came from a *series* combination of $2C$ and the other caps).
Since I know charge on $2C$, it makes sense to rewrite the energy equation in terms of charge.

$$U_2 = \frac{1}{2}Q_2\Delta V_2 = \frac{Q_2^2}{2C_2}$$

$$U_2 = \frac{\left(\frac{10}{7}\mathcal{E}C\right)^2}{2(2C)}$$

$$U_2 = \frac{\frac{100}{49}}{4}\mathcal{E}^2 C$$

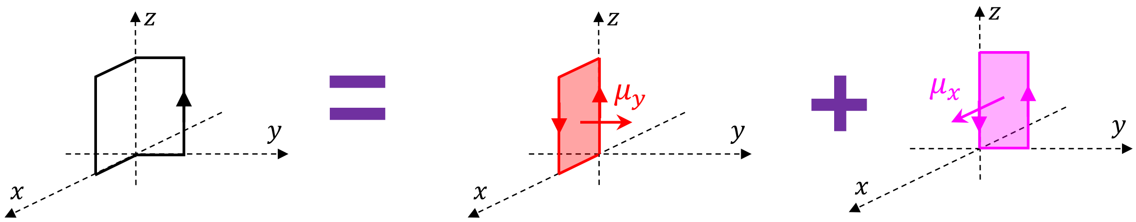
$$U_2 = \frac{25}{49}\mathcal{E}^2 C$$

$$\mathcal{E} = \sqrt{\frac{49U_2}{25C}}$$

$$\boldsymbol{\mathcal{E} = \frac{7}{5}\sqrt{\frac{U_2}{C}}}$$

$$\boldsymbol{\mathcal{E} = 1.400\sqrt{\frac{U_2}{C}}}$$

8a) I find it easiest to split the loop into two pieces (similar to workbook problem **28.16**).



Notice the currents on the z-axis for each ½-loop run in opposite directions…the sum is equivalent to the original… For each ½-loop, curl your fingers (of right hand) in the direction of current; right thumb points in direction of mag moment.

$$\vec{\mu}_{total} = \mu_x(+\hat{\imath}) + \mu_y(+\hat{\jmath})$$

8b) You are given the output of a vector equation (resultant magnitude) and asked to find an input. Follow the same procedure as always for determining a vector magnitude but use a variable for the unknown quantity (in this case i).

$$\vec{\tau} = \vec{\mu}_{total} \times \vec{B}_{ext}$$

$$\vec{\tau} = (\mu_x\hat{\imath} + \mu_y\hat{\jmath}) \times B_{ext}\hat{k}$$

$$\vec{\tau} = \mu_y B_{ext}(+\hat{\imath}) + \mu_x B_{ext}(-\hat{\jmath})$$

In this special case, we know

$$\mu_x = \mu_y = NiA = \frac{Nis^2}{2}$$

Plug in and factor out:

$$\vec{\tau} = \frac{Nis^2 B_{ext}}{2}(\hat{\imath} - \hat{\jmath})$$

Now get the magnitude…

$$\tau = \|\vec{\tau}\| = \frac{\sqrt{2}Nis^2 B_{ext}}{2}$$

Solve for current. When plugging in numbers, watch those units!!!

$$\boldsymbol{i = 542\ \mathrm{mA}}$$

MOST COMMON ISSUES:

1) Just because $\vec{\tau} = \vec{\mu} \times \vec{B}_{ext}$ one cannot say $\tau = \mu B_{ext}$.
   Get the full vector for $\vec{\mu}$, do the cross-product, *then* take the magnitude!
2) In this problem, the area of the loop before bending is s^2.
   HOWEVER, the area VECTOR is $\vec{A} = \frac{s^2}{2}\hat{\imath} + \frac{s^2}{2}\hat{\jmath}$.
   Notice the magnitude of the area vector is $A = \sqrt{\left(\frac{s^2}{2}\right)^2 + \left(\frac{s^2}{2}\right)^2} = \frac{\sqrt{2}}{2}s^2$.

9a) The force equation for *magnetic* forces is $\vec{F} = q\vec{v} \times \vec{B}_{ext}$.
Due to the cross product, this force is necessarily perpendicular to velocity (regardless of the orientation of $\vec{B}_{ext}$)!
A magnetic force can change the direction of charge velocity, but not the size of it!!!

The force equation for *electric* forces is $\vec{F} = q\vec{E}$.
Notice this can be in any direction relative to charge velocity (parallel, perpendicular, or both).
In this special case, electron between *vertically* oriented plates, electric field & force point horizontally.
In this case, $\vec{E}$ is *anti-parallel* while $\vec{F} = -e\vec{E}$ force is *parallel*.
We require a force component parallel to velocity force if speed is increasing…here only $\vec{E}$ can provide that.

9b) From the previous question, we know the electric force is tangent to the direction of motion.
Therefore the deflection force must arise from only the magnetic field (in this special case).

$$\vec{F} = q\vec{v} \times \vec{B}_{ext}$$

WATCH OUT! Charge is negative (using an electron).

$$\vec{F} = (-e)\vec{v} \times \vec{B}_{ext}$$

Furthermore we know deflection force is out of the page ($\vec{F} = F\hat{k}$) while the charges moves to the right ($\vec{v} = v\hat{\imath}$).
Using a bit of trial and error gives

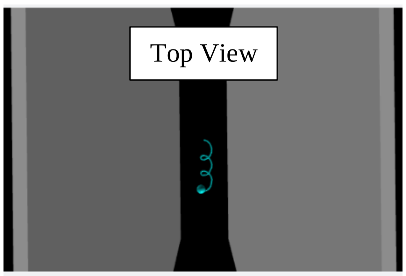
$$\vec{B}_{ext} = B_{ext}(-\hat{\jmath})$$

9c) The path would be circular if no electric field was present.
The path would be a straight line if no magnetic field was present.
The path would be parabolic if no magnetic field was present AND charge motion was perpendicular to field.
None of these conditions are met!
The best answer is none of the other answers is correct.

9d) The simplest way to get a good understanding of the trajectory is to write a simulation.
Our standard method from lab uses the Euler-Cromer Method.
Obviously there would be declaring a bunch of constants (including computing $\vec{E}$ from givens ΔV & d).
I would compute electric force *outside* of the loop (since $\vec{E}$ and thus $\vec{F}_E$ is constant between the plates).
Inside the loop I would first *compute* both $\vec{F}_B = q\vec{v} \times \vec{B}_{ext}$ and $\vec{F}_{NET} = \vec{F}_E + \vec{F}_B$.
Then *update* momentum ($ball.momentum \mathrel{+}= \vec{F}_{NET} \times dt$)…comes from $\vec{F}_{NET} = \frac{d\vec{p}}{dt}$.
Next *update* position ($ball.pos \mathrel{+}= ball.\frac{momentum}{ball.mass} \times dt$)…comes from $\vec{r} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{\vec{p}}{m}\right)$.

Note: if you code it up, the hardest part comes when thinking about the sizes of all the numbers. Think of all those wildly varying powers of 10 on all the constants. Doing some paper and pencil work to figure out the rough sizes of all the numbers is potentially useful in getting ballpark numbers for a required dt.

I initially ran my loop for about 5 time steps with print statements to check my computations. From there I revised my constants until the simulation was both visible and running at a decent speed.



At right is a screen shot I made using $\Delta V = 500\ \text{nV}, d = 1\ \text{m}, B_{ext} = 10\ \text{nT},\ v = 100\frac{\text{m}}{\text{s}}$, $dt = 10\ \text{ns}$, & sim_speed=1. While this speed is ridiculously low, I can easily scale up the speed while scaling fields commensurately. Once done, I could analyze more realistic problems. Every set of numbers gives a different trajectory!