**AFTER I GIVE THE SIGNAL TO BEGIN YOU CAN REMOVE THIS SHEET. DO NOT TURN IT IN!**

**163fa23t2a –** Once the exam has officially started, remove the top sheet. The remaining sheets comprise your exam. It is each student's individual responsibility to ensure the instructor has received her or his completed exam. Any exams not received by the instructor earn zero points. Smart watches, phones, or other devices (except scientific calculators) are not permitted during the exam.

$e = 1.602\times10^{-19}\ \mathrm{C}$ $\quad k = 8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}$ $\quad c = 3.00\times10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$ $\quad \varepsilon_0 = 8.85\times10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}$

$h = 6.626\times10^{-34}\ \mathrm{J\cdot s}$ $\quad hc \approx 1240\ \mathrm{eV\cdot nm}$ $\quad \mu_0 = 4\pi\times10^{-7}\ \frac{\mathrm{T\cdot m}}{\mathrm{A}}$ $\quad 1\ \mathrm{eV} = 1.602\times10^{-19}\ \mathrm{J}$

$m_p = 1.673\times10^{-27}\ \mathrm{kg}$ $\quad m_e = 9.11\times10^{-31}\ \mathrm{kg}$

$\vec{F} = q\vec{E}$ $\quad k = \frac{1}{4\pi\varepsilon_0}$ $\quad \Delta x = v_{ix}t + \frac{1}{2}a_x t^2$ $\quad v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$

$\vec{F}_{1on2} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{1to2}$ $\quad \vec{E} = \frac{kq}{r^2}\hat{r}$ $\quad V = \frac{kq}{r}$ $\quad U_{12} = \frac{kq_1q_2}{r_{12}}$

$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ $\quad q_{enc} = \int \rho dV$ $\quad E_{\parallel\,plates} = \frac{|\Delta V|}{d} = \frac{\sigma}{\varepsilon_0}$ $\quad E_{plate} = \frac{\sigma}{2\varepsilon_0}$

$E_{ring} = \frac{kQz}{(R^2+z^2)^{3/2}}$ $\quad V_{ring} = \frac{kQ}{(R^2+z^2)^{1/2}}$ $\quad E_x = -\frac{dV}{dx}$ $\quad V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s}$

$\Delta U = q\Delta V$ $\quad U_C = \frac{1}{2}Q_C\Delta V_C$ $\quad Q_C = \Delta V_C C$ $\quad I_C = -C\frac{dV_C}{dt}$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$ $\quad C_{eq} = C_1 + C_2 + \cdots$ $\quad C_{plates} = \frac{\varepsilon_0 A}{d}$ $\quad C' = \kappa C$

$R_{eq} = R_1 + R_2 + \cdots$ $\quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$ $\quad R = \frac{\rho L}{A}$ $\quad \rho = \rho_0(1 + \alpha\Delta T)$

$\Delta V_R = I_R R$ $\quad \mathcal{P}_R = I_R\Delta V_R$ $\quad X(t) = X_f + (X_i - X_f)e^{-t/\tau}$ where $\tau = RC$ or $\frac{L}{R}$

$\vec{F} = q\vec{v}\times\vec{B}_{ext}$ $\quad \vec{F} = I\int d\vec{s}\times\vec{B}_{ext}$ $\quad \vec{\tau} = \vec{\mu}\times\vec{B}_{ext}$ $\quad \vec{\mu} = NI\vec{A}$

$U = -\vec{\mu}\cdot\vec{B}_{ext}$ $\quad B_{sol} = \frac{\mu_0 NI}{L}$ $\quad B_{ring} = \frac{\mu_0 Ir^2}{2(r^2+z^2)^{3/2}}$ $\quad B_{straight} = \frac{\mu_0 I}{2\pi a}$

$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$ $\quad I_{enc} = \int \vec{J}\cdot d\vec{A}$ $\quad \vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}$ $\quad \Phi_B = \int \vec{B}\cdot d\vec{A}$

$EMF = -N\frac{d}{dt}\Phi_B$ $\quad EMF = B_\perp Lv$ $\quad L = \frac{\Phi_B}{I}$ $\quad U_L = \frac{1}{2}LI^2$

$\frac{\Delta V_2}{\Delta V_1} = \frac{N_2}{N_1}$ $\quad \Delta V_L = -L\frac{dI_L}{dt}$ $\quad Z = \sqrt{R^2 + (X_L - X_C)^2}$ $\quad \tan\phi = \frac{X_L - X_C}{R}$

$X_L = \omega L$ $\quad X_C = \frac{1}{\omega C}$ $\quad V_{source} = V_0\sin\omega t$ $\quad i = i_{max}\sin(\omega t - \phi)$

$\Delta V_{Rmax} = i_{max}R$ $\quad \Delta V_{Lmax} = i_{max}X_L$ $\quad \Delta V_{Cmax} = i_{max}X_C$ $\quad V_{source\ max} = i_{max}Z$

$\Delta V_{max} = \frac{\Delta V_{pk-pk}}{2}$ $\quad \Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}}$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$

$\mathcal{P}_{avg} = I_{rms}\Delta V_{rms}\cos\phi = I_{rms}^2 R$

$c = f\lambda$ $\quad k = \frac{2\pi}{\lambda}$ $\quad \omega = 2\pi f = \frac{2\pi}{\mathbb{T}}$

$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$ $\quad I_{avg} = S_{avg} = \frac{E_{max}B_{max}}{2\mu_0} = \left(\frac{1}{c}\right)\frac{E_{max}^2}{2\mu_0} = c\frac{B_{max}^2}{2\mu_0}$

$\frac{E_{max}}{B_{max}} = c$ $\quad E_\gamma = hf = \frac{hc}{\lambda}$

$Rad.Pressure = \frac{Force}{Area} = \frac{S_{avg}}{c}$ $\quad Photon\ momentum = p_\gamma = \frac{E_\gamma}{c}$

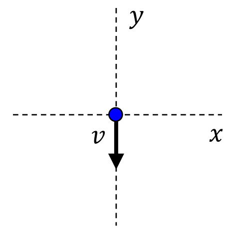
| **Material** | **Resistivity** at 20° C (in SI units) | **Temp. Coefficient** (in SI units) |
|---|---|---|
| Silver | $1.62\times10^{-8}$ | $4.1\times10^{-3}$ |
| Copper | $1.69\times10^{-8}$ | $4.3\times10^{-3}$ |
| Aluminum | $2.75\times10^{-8}$ | $4.4\times10^{-3}$ |
| Nichrome | $1.00\times10^{-6}$ | $0.4\times10^{-3}$ |
| Carbon | $3.5\times10^{-5}$ | $-0.5\times10^{-3}$ |
| Germanium | 0.46 | $-48\times10^{-3}$ |

| | |
|---|---|
| $\int\frac{x\,dx}{(x^2+a^2)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$ | $\int\frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert$ |
| $\int\frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}} = \frac{1}{a^2}\sin\theta$ | $\int\frac{x\,dx}{\sqrt{x^2\pm a^2}} = \sqrt{x^2\pm a^2}$ |
| $\int\frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$ | $\int\sqrt{x^2\pm a^2}\,dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \pm \frac{a^2}{2}\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert$ |
| $\int\frac{x\,dx}{x^2+a^2} = \frac{1}{2}\ln\lvert x^2+a^2\rvert$ | **Binomial expansion:** $(1\pm\delta)^n \approx 1 \pm n\delta + \cdots$ |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\mathrm{T} = 10^{12}$ | $\mathrm{G} = 10^{9}$ | $\mathrm{M} = 10^{6}$ | $\mathrm{k} = 10^{3}$ | $\mathrm{c} = 10^{-2}$ | $\mathrm{m} = 10^{-3}$ | $\mu = 10^{-6}$ | $\mathrm{n} = 10^{-9}$ | $\mathrm{p} = 10^{-12}$ | $\mathrm{f} = 10^{-15}$ | $\mathrm{a} = 10^{-18}$ |

**Rip off the eqt'n sheet and put your name on this page.** **NAME:___________________________________**

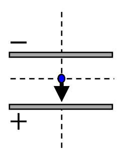
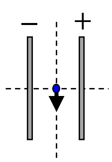
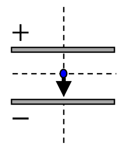
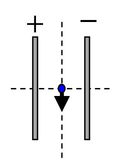
An *electron* moves *downwards* with speed $v$. The electron is in the presence of a uniform external magnetic field. The electron experiences magnetic force to the *into the page* at the instant shown.




***1a) Which best describes the signs of field components? Circle the best answers.

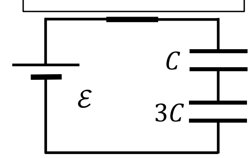
| | | | |
|---|---|---|---|
| $B_x > 0$ | $B_x = 0$ | $B_x < 0$ | Impossible to determine without more info |
| $B_y > 0$ | $B_y = 0$ | $B_y < 0$ | Impossible to determine without more info |
| $B_z > 0$ | $B_z = 0$ | $B_z < 0$ | Impossible to determine without more info |

1b) Suppose one wanted to counteract this magnetic deflection force by putting the system between charged parallel plates. Which of the following orientations is best suited to counteract the magnetic force? Circle best answer.
Note: assume the plates are infinitely large with a cross-section indicated. Plate polarity is indicated in each figure.

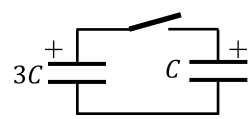
| | | | | | |
|---|---|---|---|---|---|
|  |  |  |  | It is possible, but none of the previous answers is correct | Impossible to determine without more information |

Two capacitors are connected in series to a battery and allow to fully charge (see charging circuit). After charging, the capacitors are reconnected as shown in the "Reconnected Circuit" figure at right.

Charging circuit




Reconnected circuit




2a) After initial charging which cap store more energy?

| | | | |
|---|---|---|---|
| $U_C > U_{3C}$ | $U_C = U_{3C}$ | $U_C < U_{3C}$ | Impossible to determine without more information |

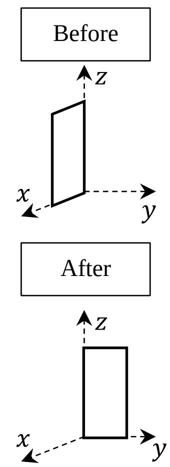
2b) The switch in the Reconnected Circuit is closed and the capacitors are allowed to reach equilibrium.
Which of the following statements best describes <u>after the reconnected circuit is allowed to reach equilibrium</u>?

| | | | | |
|---|---|---|---|---|
| $C$ & $3C$ carry the *same charge* as each other. | $C$ & $3C$ have the *same potential difference* as each other. | $C$ & $3C$ store the *same energy*. | More than one of the previous answers is correct. | Impossible to determine without more information |

2c) The switch in the Reconnected Circuit is closed and the capacitors are allowed to reach equilibrium.
Which of the following statements are true <u>after the reconnected circuit is allowed to reach equilibrium</u>?

| | | | |
|---|---|---|---|
| Potential difference *decreases* across both capacitors. | Potential difference increases for one capacitor but decreases for the other. | Potential difference *increases* across both capacitors. | Impossible to determine without more information |

A rectangular coil of wire is supported by a hinge on the $z$-axis. The rectangular coil is twice as tall as it is wide and is in the $xz$-plane. Current in the coil flows *downwards* along the $z$-axis.




Suppose we want to introduce a magnetic torque which cause the loop to rotate towards the $yz$-plane as shown in the “After” picture. To be clear, it is ok if the loop continues swinging past the state shown in the “After” picture. We are able to apply a uniform magnetic field along any of the principal directions.

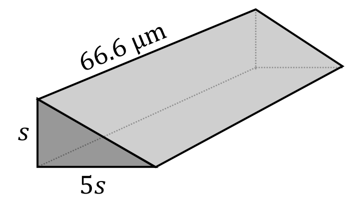
3a) What direction best describes the initial state magnetic moment of the loop?

<table>
<tr><td>$+\hat{\imath}$</td><td>$+\hat{\jmath}$</td><td>$+\hat{k}$</td><td rowspan="2">Some combo of $\pm\hat{\imath}$ & $\pm\hat{\jmath}$</td><td rowspan="2">Some combo of $\pm\hat{\jmath}$ & $\pm\hat{k}$</td><td rowspan="2">Some combo of $\pm\hat{k}$ & $\pm\hat{\imath}$</td><td rowspan="2">Impossible to determine without more info</td></tr>
<tr><td>$-\hat{\imath}$</td><td>$-\hat{\jmath}$</td><td>$-\hat{k}$</td></tr>
</table>

3b) What direction should be used for the external magnetic field to cause the desired rotation?

<table>
<tr><td>$+\hat{\imath}$</td><td>$+\hat{\jmath}$</td><td>$+\hat{k}$</td><td rowspan="2">Requires some combo of $\pm\hat{\imath}$ & $\pm\hat{\jmath}$</td><td rowspan="2">Requires some combo of $\pm\hat{\jmath}$ & $\pm\hat{k}$</td><td rowspan="2">Requires some combo of $\pm\hat{k}$ & $\pm\hat{\imath}$</td><td rowspan="2">Impossible to determine without more info</td></tr>
<tr><td>$-\hat{\imath}$</td><td>$-\hat{\jmath}$</td><td>$-\hat{k}$</td></tr>
</table>

An engineer wishes to make a resistor using a right triangular prism of carbon. When the resistor is in operation in a furnace she wants a resistance of 333 nΩ between the front & back triangular faces (separated by the distance shown in the figure at right, not to scale).
**The operating temperature is expected to be 144℃.**
****4a) What value is required for dimension $s$?

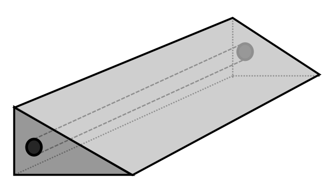



| | |
|---|---|
| 4a | |

4b) During a transient heating cycle, the furnace temporarily operates at 180℃.
How is the original resistance affected by operating at warmer than expected temperatures?

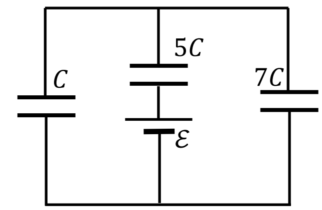
| | | | |
|---|---|---|---|
| Decreases $R$ | No effect on $R$ | Increases $R$ | Impossible to determine without more info. |

4c) After the resistor is made, the engineer considers drilling a hole as shown at right.
How would drilling this hole affect the resistor’s resistance?



| | | | |
|---|---|---|---|
| Decreases $R$ | No effect on $R$ | Increases $R$ | Impossible to determine without more info. |

Consider the capacitor circuit shown at right.
All capacitances & the battery potential difference are known.

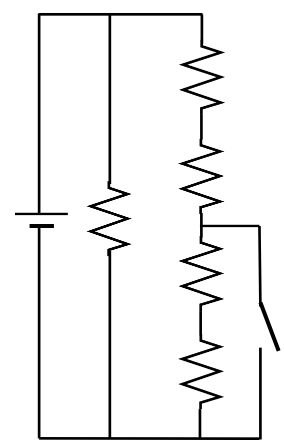



**5a) Determine equivalent capacitance in terms of $C$.

**5b) Rank the charges stored by each capacitor (clearly indicating any ties).
I expect your answer to look like $Q_1 = Q_5 > Q_7$ or something like that.

| | |
|---|---|
| 5a | |
| 5b | |

In the circuit shown at right a $6.00$ V battery pushes $1.00$ A current when the switch is *open*.
All resistors have equal resistance. To be clear, the current in the battery's branch is $1.00$ A.



**6a) Determine the resistance of a single resistor.

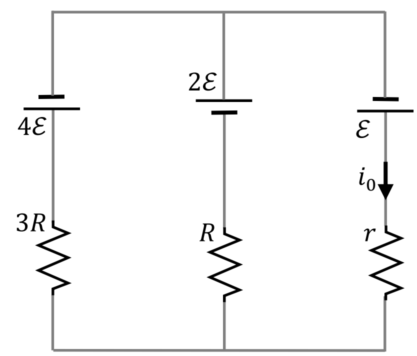
***6b) By what *factor* does total power delivered change when the switch is *closed*?
Express this factor as a number with three sig figs.

| | |
|---|---|
| 6a | |
| 6b | |

In the circuit shown at right resistance $r$ is UNKNOWN.
Current $i_0$, the voltages, and the other resistances are known.

***7a) Write a linearly independent set of loop and junction equations one could use to analyze this circuit. **To ensure full credit, clearly label the figure at right with additional currents and loop directions used for your equations.**

***7b) Determine resistance $r$ in terms of $\mathcal{E}, R,$ & $i_0$.
Simplify your work a reasonable amount for full credit.



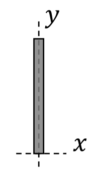

| **Loop Equations** | **Junction Equations** |
| --- | --- |
| | |

| | |
| --- | --- |
| 7b | |

A straight wire segment aligned with the $y$-axis of length $2d$ carries current $I$.
A non-uniform external magnetic field is present given by

$$\vec{B} = \frac{\beta}{(y^2 + d^2)^{1/2}}\hat{k}$$

where $\beta$ is a positive constant.




8a) Which best describes the *direction* of the magnetic force acting on the straight wire segment?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $+\hat{\imath}$ | $+\hat{\jmath}$ | $+\hat{k}$ | Some combo of $\pm\hat{\imath}$ & $\pm\hat{\jmath}$ | Some combo of $\pm\hat{\jmath}$ & $\pm\hat{k}$ | Some combo of $\pm\hat{k}$ & $\pm\hat{\imath}$ | Impossible to determine without more info | None of the other answers is correct |
| $-\hat{\imath}$ | $-\hat{\jmath}$ | $-\hat{k}$ | | | | | |

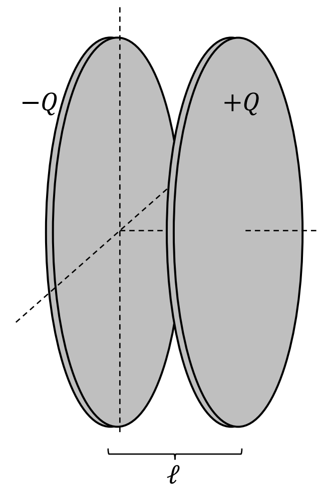
8b) Determine the units assumed on $\beta$.

****8c) Determine the *magnitude* of the magnetic force exerted on the wire segment.

| | |
|---|---|
| 8b | |

| | |
|---|---|
| 8c | |

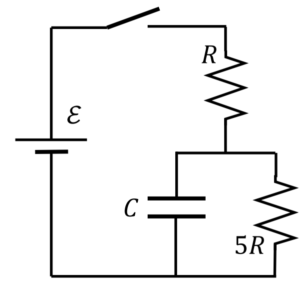
Two parallel plates are arranged as shown in the figure. You may assume the radius $R$ of a single plate is very large compared to the plate spacing $\ell$ (that letter is "script L").




****9) Derive capacitance between the two parallel plates.
Note: credit will only be given if your derivation is clear and complete.
That said, please put your final result in the box as well.

| | |
|---|---|
| 9 | |

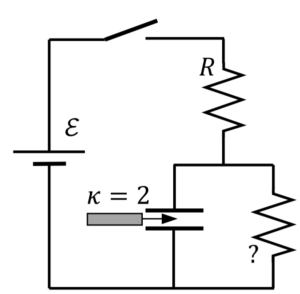
Consider the circuit shown at right. Assume $\mathcal{E}, R, \&\ C$ are known.
Assume the switch is closed at time $t = 0$.
10a) Determine initial current through $R$ *just after* the switch is closed.
10b) Determine steady-state current through $R$ (*a long time after* the switch is closed).
10c) Determine steady-state voltage across the capacitor.
*****10d) Suppose the switch is re-opened. Determine time required (after re-opening) for capacitor *energy* to decrease by 33.3% from its max value. <u>Answer as decimal number with 3 sig figs times $RC$.</u>
**Notice 10e) At bottom of the page…**




| | |
|---|---|
| 10a | |
| 10b | |
| 10c | |
| 10d | |

10e) An engineer builds a nearly identical circuit but inserts a dielectric with $\kappa = 2$ into the capacitor. The engineer wants the same discharge rate upon re-opening the switch. Can it be done? If so, how? Circle the best answer.

| | | | | |
|---|---|---|---|---|
| Change $5R$ to $10R$ | No change to $5R$ is required | Change $5R$ to $2.5R$ | Impossible to adjust $5R$ to get the same discharge rate | Impossible to determine without more info |



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