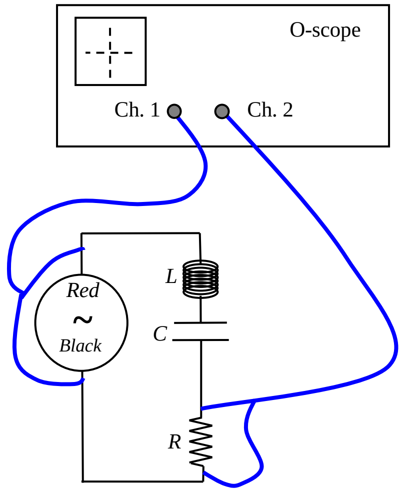
Series LRC Circuit



Apparatus: function generator, BNC to alligator clip cable, O-scopes, TWO probes per table, LCR meter, breadboards (but no jumper cables), DMMs and probe leads

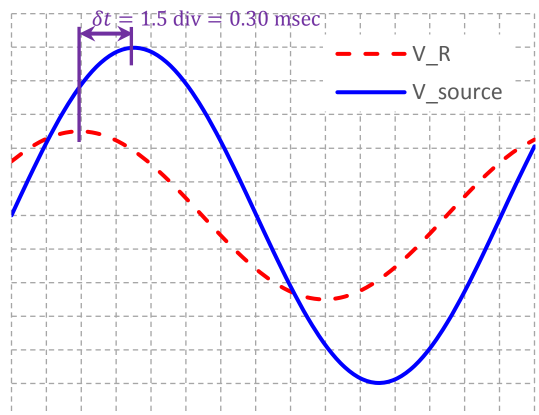
Each lab group requires one *each* of the following circuit components:

- 10.0 mH inductor
- 1.0 mH inductor
- 1 µF ceramic capacitor
- 1.0 kΩ resistor
- 470 Ω resistor

Goal:

Pair up with another lab group.
One lab group must use the large inductor, the other group the small inductor.
Assemble the series LRC circuit shown.
Simultaneously measure $V_{source\ pk-pk}$ and $V_{R\ pk-pk}$ for a particular source frequency.
While keeping source *voltage* fixed, sweep source *frequency*.
For each frequency, tabulate current mplitude (i_{max}) and phase angle (ϕ).
Make plots showing theoretical and experimental values (use Sample Graph Type II formatting).
Make the horizontal axis logarithmic.
Repeat all this for a second LRC circuit using a different value of R.
At the end, share your two plots with the other group.

This section describes how to get experimental data from a scope.
A typical scope screen might look a bit like the plot at right.
Assume each horizontal division is 0.2 msec.
Assume each vertical division is 2.0 V.



Source (pk-to-pk) voltage is 10 vertical divisions ($V_{source\ pk-pk} = 20.0$ V).
Peak-to-peak voltage across the resistor is *about* half that ($V_R = 10.0$ V).
The period is about 14 horizontal divisions ($\mathbb{T} = 2.8$ msec).
The frequency is $f = \frac{1}{\mathbb{T}} = \frac{1}{2.8\times10^{-3}\ \text{s}} = 357$ Hz.

The resistor peaks just before the source.
Recall current is given by $\frac{V_R}{R}$.
Therefore, $i(t)$ leads $V_{source}(t)$ if resistor voltage leads source voltage!
Here current leads the source voltage (or source voltage lags current).
This is a *capacitively* dominated circuit (ELI the ICE man).
We expect the phase angle should be *negative*.

The time *between the two peaks* is about 1.5 horizontal divisions ($t = 0.30$ msec).
The phase can be determined using a ratio $\frac{\delta t}{\mathbb{T}} = \frac{\phi}{360°}$ This gives

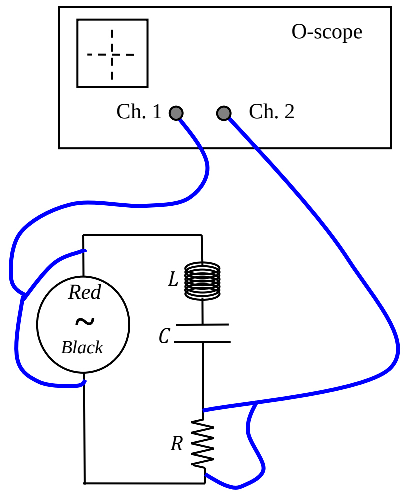
$$\phi = 360°\frac{\delta t}{\mathbb{T}} = 360°\frac{(0.30\ \text{msec})}{(2.8\ \text{msec})} = 360°\frac{(1.5\ \text{boxes})}{(14\ \text{boxes})} = -39°$$

If $\underline{i(t)}$ leads $\underline{V_{source}(t)}$ we assume $\underline{\phi}$ is negative; the circuit is capacitively dominated (ELI the ICE man).
Note: for our circuits all phase angles should be between $-90°$ & $90°$.

To determine <u>theoretical</u> values, we will use the following set of equations:

Relating *peak-to-peak* voltage, voltage *amplitude*, and RMS voltage	$V_{max} = \frac{V_{source\ pk-pk}}{2} = \sqrt{2}V_{RMS}$
Relating peak source *voltage* to peak *current*	$i_{max} = \frac{V_{source\ max}}{Z} = \frac{V_o}{Z}$
Total *Impedance* (in units of Ω)	$Z = \sqrt{R^2 + (X_L - X_C)^2}$
Phase angle typically expressed in *degrees* (convert to *radians* for computations)	$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$
Capacitive *Reactance* (in units of Ω)	$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$
Inductive *Reactance* (in units of Ω)	$X_L = \omega L = 2\pi f L$
WATCH OUT!!! In the previous two formulas, the symbol f stands for *source* frequency (the frequency of the function generator). It IS NOT *resonance* frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$.	
The equations listed below are *not* necessary to complete this lab. I am including them as it may be helpful to your understanding of the series LRC circuit.	
Source voltage as function of time	$V_{source}(t) = V_{source\ max}\sin(\omega t)$
Current as a function of time Notice a positive phase angle	$i(t) = i_{max}\sin(\omega t - \phi)$
WATCH OUT!!! In many resources you will see $i(t) = i_{max}\sin(\omega t)$ & $V_{source}(t) = V_{source\ max}\sin(\omega t + \phi)$. Shifting $i(t)$ *to the left* by ϕ (relative to $V_{source}(t)$) is the same as shifting $V_{source}(t)$ to the right by ϕ. **WATCH OUT!!!** In the previous two formulas, the symbol $\omega = 2\pi f$ stands for *source* frequency (the frequency of the function generator). **WATCH OUT!!!** People often say source *frequency* for ω even though it is actually source *angular frequency*. In general, people use the units ($\frac{\text{rad}}{\text{s}}$ versus Hz) to identify if you mean ω or f…	
Voltage *amplitude* across the capacitor	$V_{C\ max} = i_{max}X_C$
Voltage across the capacitor *as a function of time* Hint: ELI the ICE man…$V_C(t)$ must *lag* current…thus the extra $-\frac{\pi}{2}$	$V_C(t) = V_{C\ max}\sin(\omega t - \phi - \frac{\pi}{2})$
Voltage *amplitude* across the inductor	$V_{L\ max} = i_{max}X_L$
Voltage across the inductor *as a function of time* Hint: ELI the ICE man…$V_L(t)$ must *lead* current…thus the extra $+\frac{\pi}{2}$	$V_L(t) = V_{L\ max}\sin(\omega t - \phi + \frac{\pi}{2})$

Reminder of what you are supposed to get done:



- Simultaneously measure the function generator ($V_{source\ pk-pk}$) and the voltage across the resistor (V_R) for a particular source frequency.
- While keeping source *voltage* fixed, sweep source *frequency*.
- For each frequency, tabulate peak current (i_{max}) and phase angle (ϕ).
- Repeat all this for a second LRC circuit using a different value of R.
- At the end, swap plots with another group so each group has 2 plots for 4 distinct cases of L, R, and C.

To get data:

- Be sure the output cable of the function generator is connected to the **output** BNC connector!
- Ensure you are on a sine wave and make the generator produce with a 1 kHz sine wave.
- Ensure DC offset button is NOT depressed.
- Set the amplitude to a convenient size (say 10.0 V amplitude). Before proceeding, verify the wave amplitude is 10.0 V by doing the following:
  - Connect the function generator directly to Ch 1 on the scope (without any resistors, etc).
  - Hit AutoRange button if it is not already lit up green.
  - Use the MEASURE button so the scope measures frequency, period, and peak-to-peak voltage.
  - Adjust the output know of the function generator until you hit 20.0 V peak-to-peak.
    - Dividing peak-to-peak voltage by 2 is more reliable than having the scope measure peak voltage (e.g. if there is DC offset).
- Wire up the circuit as shown on in the figure at the top of this page.
  - Please ensure that one leg of the resistor is connected to the black lead of the function generator!
- Determine your *target* frequencies.
  - These are the *approximate* values of you will shoot for when tuning the function generator.
  - When doing the actual experiment, you tune the dial *approximately* to these frequencies.
  - The function generator uses f while the most equations use ω.
  - Function generator frequencies are in units of Hz.

$0.01f_0$	$0.02f_0$	$0.05f_0$	$0.1f_0$	$0.2f_0$	$0.5f_0$	$0.75f_0$	f_0	$1.25f_0$	$2f_0$	$5f_0$	$10f_0$	$20f_0$	$50f_0$

- Do a quick sweep of the frequencies without taking any data to ensure your circuit is working properly.
  - At very low (< 50 Hz) or very high frequencies (> 500 kHz) we expect almost no current (negligible voltage across the resistor). Use the AutoRange & MEASURE buttons to verify this
  - Next, tune the function generator near f_0 on the function generator. Verify the current is large (voltage across the resistor should be large.
- Remember we will measure $V_{R\ max}$ using

$$V_{R\ max} = \frac{V_{R\ pk-pk}}{\mathbf{2}}$$

  This avoids having to think about any DC offset caused by the function generator.
- Now use the scope's MEASURE button to record the *actual* operating frequency and $V_{R\ pk-pk}$ near each target frequency. With the exception of resonance, you needn't hit the target frequency exactly…
  - In your data table, record the *actual* operating frequencies measured in the experiment!
  - $i_{max\ th} = \frac{V_{source\ max}}{Z}$ (calculate using *measured* values of $L, C, R, f, \&\ V_{source\ max}$)
  - $i_{max\ exp} = \frac{V_{R\ max}}{R} = \frac{V_{R\ pk-pk}}{2R}$ (voltage *amplitude* across R over the measured value of resistance)

Tabulate i_{max} vs. ω for both sets of data.

Create a column of theoretical data. Use your equation for Z and the function generator voltage *amplitude* to determine an equation for $i_{max\ th}$ using the *measured* values of $R, L, C, V_{source\ max}$, and ω.
WATCH OUT! When using spreadsheet formulas, be sure you properly account for prefixes!!!

Plot i_{max} vs. ω for both theory and experiment on the same chart. Repeat for the second set of data. This graph should look like Sample Graph Type II in the appendices. Use a logarithmic scale for the ω-axis by right-clicking that axis and selecting "format axis". Look around for the Logarithmic check-box. Show the theory as a smooth line with no points and show the experiment as points only with no smooth line. Since you have more than one set of data on the same graph you'll need a legend to explain the various curves.
Now making plots:

- Make the horizontal axis logarithmic for each case.
- Make a plot of i_{max} vs ω including both theoretical and experimental values (use Sample Graph Type II formatting). You might consider trying to put all 4 experimental and all 4 theoretical curves in one plot. If too hard to read, perhaps plot 1 or 2 data sets per plot.
- Make a plot of ϕ vs ω including both theoretical and experimental values (use Sample Graph Type II formatting). You might consider trying to put all 4 experimental and all 4 theoretical curves in one plot. If too hard to read, perhaps plot 1 or 2 data sets per plot.

Conclusion Questions:
1) Suppose you wanted to decrease the resonance frequency. You have only one resistor and one inductor but you do have a second identical capacitor. Would you want to put the capacitor in series or parallel with the first capacitor to decrease the resonant frequency? Explain why.

2) As resistance in the circuit decreases, will the resonant frequency increase, decrease, or stay the same? Explain.

3) Consider the plot of i_{max} vs ω. Generally speaking, it looks a bit like a hill or mountain. What happens to the shape of this hill/mountain as resistance decreases? Discuss both hill/mountain height and width. Consider reading workbook problem **31.5** carefully…

4) The function generator has $50\ \Omega$ of internal impedance not accounted for in our theory. Consider the peak height in each of your plots of i_{max} versus ω. Approximately what % error is introduced by ignoring the $50\ \Omega$ internal impedance for each circuit ($R_1 \approx 1$ kΩ and $R_2 \approx 500\ \Omega$)? Compare these results to the difference between your experimental and theoretical values of i_{max} at resonance.

Note: ignoring the $50\ \Omega$ of internal impedance has no effect on our *measurement* of i_{max}. This is because we used

$$i_{exp\ max} = \frac{V_{R\ max}}{R}$$

Here $V_{R\ max}$ is voltage across *the resistor* while R is *resistance* of only that resistor (not total impedance Z).

This is a good trick to remember. Whenever you need current in a circuit branch, you can always measure voltage across a resistor in that branch (and divide by R) to get current.