Chapter 33 Solutions

33.1 Don't overthink it. When flying through empty space (or air) the speed of light is constant at approximately

$$c = 3.00 \times 10^8 \, \frac{\mathrm{m}}{\mathrm{s}}$$

$$distance = rate \cdot time$$

$$distance = \left(3.00 \times 10^8 \, \frac{\mathrm{m}}{\mathrm{s}}\right) \cdot (1.00 \times 10^{-9} \, \mathrm{s}) = 30 \, \mathrm{cm}$$

Similarly, the time to travel one meter is about 3.33 ns.

33.2 Resonance frequency of an LC-oscillator is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(3.00 \times 10^{-6} \, \mathrm{H})(175 \times 10^{-12} \, \mathrm{F})}} = 6.9\underline{4}6 \, \mathrm{MHz}$$

The photon energy is thus

$$E_\gamma = hf$$

$$E_\gamma = \frac{h}{2\pi\sqrt{LC}}$$

$$E_\gamma = \frac{(6.626 \times 10^{-34} \, \mathrm{J \cdot s})}{2\pi\sqrt{(3.00 \times 10^{-6} \, \mathrm{H})(175 \times 10^{-12} \, \mathrm{F})}}$$

$$E_\gamma = 4.6\underline{0}2 \times 10^{-27} \, \mathrm{J}$$

It is common to convert this photon energy to eV's using $1.602 \times 10^{-19} \, \mathrm{J} = 1 \, \mathrm{eV}$.

$$E_\gamma = 2.8\underline{7}3 \times 10^{-8} \, \mathrm{eV} = 28.7 \, \mathrm{neV}$$

Using Wikipedia I classified this wave as a high frequency radio wave.

33.3 Assuming the microwaves are propagating through empty space

$$c = f\lambda$$

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \, \frac{\mathrm{m}}{\mathrm{s}}}{2.45 \times 10^9 \, \frac{1}{\mathrm{s}}} = 12.2 \, \mathrm{cm}$$

33.4 Try a web search to find some simulations.
I found searching for the terms "radiating charge" and "radiation patterns" interesting.

33.5 We are given the intensity in odd units. First convert the units

$$I = 555\frac{\text{fW}}{\text{mm}^2} = 555\times10^{-15}\frac{\text{W}}{\text{mm}^2}\times\frac{(1000\text{ mm})^2}{(1\text{ m})^2} = 5.55\times10^{-7}\frac{\text{W}}{\text{m}^2}$$

We know electric field amplitude relates to intensity using

$$I = \left(\frac{1}{c}\right)\frac{E_{max}^2}{2\mu_0}$$

$$E_{max} = \sqrt{2Ic\mu_0}$$

$$E_{max} = \sqrt{2\left(5.55\times10^{-7}\frac{\text{W}}{\text{m}^2}\right)\left(3.00\times10^{8}\frac{\text{m}}{\text{s}}\right)\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)}$$

$$E_{max} = \sqrt{2\left(5.55\times10^{-7}\frac{\text{W}}{\text{m}^2}\right)\left(3.00\times10^{8}\frac{\text{m}}{\text{s}}\right)\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)}$$

$$E_{max} = 20.4\frac{\text{mV}}{\text{m}}$$

Regarding the units:

By using SI units to start, I figure I am correct. That said, one possible way to figure them out is shown below. It was the first style that came to mind. There are probably other, faster ways to see how the units do work out.

$$[E_{max}] = \sqrt{\frac{\text{W}}{\text{m}^2}\cdot\frac{\text{m}}{\text{s}}\cdot\frac{\text{T}\cdot\text{m}}{\text{A}}}$$

$$[E_{max}] = \sqrt{\text{W}\cdot\frac{1}{\text{s}}\cdot\frac{\text{T}}{\text{A}}}$$

$$[E_{max}] = \sqrt{\frac{\text{J}}{\text{s}}\cdot\frac{1}{\text{s}}\cdot\frac{\text{T}}{\text{A}}}$$

To get A & T into more standard units consider an equation which includes A & T with something else in standard units. Force came to mind. I know $F_B = ILB\sin\theta$. This tells me units of T are equivalent to $\frac{\text{N}}{\text{A}\cdot\text{m}} = \frac{\text{kg}\cdot\frac{\text{m}}{\text{s}\text{s}}}{\text{A}\cdot\text{m}} = \frac{\text{kg}}{\text{A}\cdot\text{s}^2}$. Notice this makes $\frac{\text{T}}{\text{A}} = \frac{\text{kg}}{\text{A}^2\cdot\text{s}^2}$.

$$[E_{max}] = \sqrt{\frac{\text{J}}{\text{s}}\cdot\frac{1}{\text{s}}\cdot\frac{\text{kg}}{\text{A}^2\cdot\text{s}^2}}$$

$$[E_{max}] = \sqrt{\frac{\text{N}\cdot\text{m}}{\text{s}}\cdot\frac{1}{\text{s}}\cdot\frac{\text{kg}}{\left(\frac{\text{C}}{\text{s}}\right)^2\cdot\text{s}^2}}$$

$$[E_{max}] = \sqrt{\frac{\text{N}}{\text{C}^2}\cdot\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}$$

$$[E_{max}] = \sqrt{\frac{\text{N}}{\text{C}^2}\cdot\text{N}}$$

$$[\boldsymbol{E_{max}}] = \frac{\textbf{N}}{\textbf{C}} = \frac{\textbf{V}}{\textbf{m}}$$

33.6

a) Without computing, you are supposed to recognize high frequencies correspond to short wavelengths. Conversely, the longest wavelength in the AM band must come from the lowest possible frequency.

b) Without computing, you are supposed to recognize high frequencies correspond to high energies. The highest possible energy corresponds to the highest frequency in the AM band.

c) No. The energies are equally space because energy depends linearly on frequency ($E = hf$).

d) **Alternate style shown below:** Wavelength is inversely proportional to frequency ($\lambda = \frac{c}{f}$). Notice

$$\frac{d\lambda}{df} = -\frac{c}{f^2}$$
$$|d\lambda| = \frac{c}{f^2} df$$

In this equation $df = 10$ kHz is the frequency spacing. Notice the smallest wavelength spacing is given at high frequencies. Note: this method is only valid if $df \ll f$.

$$|d\lambda_{min}| = \frac{c}{f_{max}^2} df = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{\left(1605 \times 10^3 \frac{1}{s}\right)^2} \left(10 \times 10^3 \frac{1}{\text{s}}\right) = 1.16 \text{ m}$$

$$|d\lambda_{max}| = \frac{c}{f_{min}^2} df = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{\left(535 \times 10^3 \frac{1}{\text{s}}\right)^2} \left(10 \times 10^3 \frac{1}{\text{s}}\right) = 10.5 \text{ m}$$

Alternate style: A lot of you probably just did this

$$\lambda_{max} = \frac{c}{f_{min}} = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{535 \times 10^3 \frac{1}{\text{s}}} = 560.7 \text{ m}$$

$$\lambda_{almost\ max} = \frac{c}{f_{min} + \delta f} = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{545 \times 10^3 \frac{1}{\text{s}}} = 550.4 \text{ m}$$

$$\delta\lambda_{max} = \lambda_{max} - \lambda_{almost\ max} = 10.3 \text{ m}$$

Strictly speaking, this method is more precise than the first method. That said, the other method is great for getting an *approximate* answer quickly for a number of cases.

33.7 TRICK QUESTION. All three travel with the same *speed* (through empty space).
The *frequency* of electric field oscillations differs for the waves…not the propagation *speed*.

Note: the waves *do* travel at different speeds through *glass* or *water*.
This effect can be observed with a prism!
When the waves spread out into a rainbow of colors the physical phenomenon is called *dispersion*.
The waves spread out because of their different propagation speeds *in a medium other than vacuum or air*!

33.8

a) The energy is usually obtained using

$$E = \frac{hc}{\lambda} \quad \text{where} \quad hc = 1240\ \text{eV}\cdot\text{nm}$$

$$1\ \text{km} = 1 \times 10^3\ \text{m} = 1 \times 10^3\ (10^9\ \text{nm}) = 10^{12}\ \text{nm}$$

$$E = \frac{1240\ \text{eV}\cdot\text{nm}}{10^{12}\ \text{nm}}$$

$$\boldsymbol{E = 1.24\ \text{neV}}$$

The frequency is found using

$$E = hf \quad \text{where} \quad h = 6.626 \times 10^{-34}\ \text{J}\cdot\text{s}$$

Use $1.602 \times 10^{-19}\ \text{J} = 1\ \text{eV}$ to convert h to $4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$.

$$f = \frac{E}{h}$$

$$f = \frac{1.24 \times 10^{-9}\text{eV}}{4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}}$$

$$f = 300\ \text{kHz}$$

b) Energy and frequency relate to each other linearly.
If we have 10^{12} times the energy we also have 10^{12} times the frequency.
The new frequency is $f' = 300 \times 10^{15}$ Hz $= 300$ PHz where PHz means PetaHertz…yeah…I had to look that one up, too! The new energy is $E' = 1.24$ keV. The wave is classified as a soft x-ray.

33.9 Generally speaking, we assume the wave propagates in the same direction as the Poynting vector named after John Henry Poynting. It is a convenient misnomer to think the wave points in the direction of the Poynting vector. To be clear, the direction of the Poynting vector is the direction *energy* travels.

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

As with any cross-product, we can use a right hand rule to determine the direction of the output vector.
Align the finger s of your right hand with the first vector, curl them to the second vector, and your thumb points in the direction of the output vector. With electromagnetic waves, we sometimes use coordinate systems other than the standard. Part of this question is to remind you to always check the coordinate system.

Case 1: We see $\hat{E} = -\hat{\imath}$ & $\hat{B} = +\hat{\jmath}$. The cross-product points into the page which corresponds to $-\hat{k}$.

Case 2: We see $\hat{E} = -\hat{\jmath}$ & $\hat{B} = -\hat{\imath}$. The cross-product points downwards which corresponds to $-\hat{k}$.

33.10 We are given a plane wave propagating through empty space has electric field

$$\vec{E}(t) = \left(2.22\,\frac{\mathrm{V}}{\mathrm{m}}\right)\sin\left\{kz - \left(4.44\times10^{14}\,\frac{\mathrm{rad}}{\mathrm{s}}\right)t\right\}\hat{\jmath}$$

a) We know $E_{max} = 2.22\,\frac{\mathrm{V}}{\mathrm{m}}$ & $\omega = 4.44\times10^{14}\,\frac{\mathrm{rad}}{\mathrm{s}}$. Since propagating through free space we also know wave speed is $c = 3.00\times10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}$. Using $v = f\lambda = \frac{\omega}{k}$ & $k = \frac{2\pi}{\lambda}$ we find

$$k = \frac{\omega}{c} = 1.48\times10^{6}\ \mathrm{m}^{-1} \qquad \& \qquad \lambda = \frac{2\pi}{k} = 4.2\underline{4}5\ \mu\mathrm{m}$$

b) Look inside the sine term to see what symbol appears with k. In this case we see the wave is of the form

$$\vec{E}(t) = E_{max}\sin\{kz - \omega t\}\hat{\jmath}$$

This wave is propagating along the z-axis.

If you are curious, a more general way to write the wave equation is

$$\vec{E}(t) = E_{max}\sin\{\vec{k}\cdot\vec{r} - \omega t\}\hat{E}$$

Notice we get back our standard solutions for special cases of $\vec{r}$:

$$\text{when}\quad \vec{r} = x\hat{\imath}\quad \text{we get}\quad \vec{k}\cdot\vec{r} = k_x x$$
$$\text{when}\quad \vec{r} = y\hat{\jmath}\quad \text{we get}\quad \vec{k}\cdot\vec{r} = k_y y$$
$$\text{when}\quad \vec{r} = z\hat{k}\quad \text{we get}\quad \vec{k}\cdot\vec{r} = k_z z$$

c) When $kz - \omega t = \frac{\pi}{2}$ rad we know the electric field points $+\hat{\jmath}$.

We know the wave propagates $+\hat{k}$.

We also know the wave propagates in the direction of the Poynting vector $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$.

The output of the cross-product points $+\hat{k}$ and the first input points $+\hat{\jmath}$.

In order to make the right hand rule work, the second input to the cross-product ($\vec{B}$) points $-\hat{\imath}$.

The magnetic field oscillates parallel to the x-axis with equation

$$\vec{B}(t) = B_{max}\sin\{kz - \omega t\}(-\hat{\imath})$$

d) To relate electric field to magnetic field we use

$$c = \frac{E_{max}}{B_{max}}$$

If it helps, I memorized this equation by thinking about velocity selector problems from crossed fields.

$$B_{max} = \frac{E_{max}}{c} \approx 7.4\ \mathrm{nT}$$

PROBLEM CONTINUES ON NEXT PAGE…

e) WATCH OUT! There is a difference between AVERAGE intensity and intensity.
The AVERAGE intensity is given by the AVERAGE Poynting vector (magnitude)

$$I_{avg} = S_{avg} = \frac{E_{max}^2}{2c\mu_0} = \frac{cB_{max}^2}{2\mu_0}$$

Intensity (as a function of time) is the magnitude of the *instantaneous* Poynting vector.

$$I = \left\|\vec{S}\right\|$$

$$I = \left\|\frac{\vec{E}\times\vec{B}}{\mu_0}\right\|$$

$$I = \frac{1}{\mu_0}\left\|(E_{max}\sin\{kz-\omega t\}\hat{\jmath})\times(B_{max}\sin\{kz-\omega t\}(-\hat{\imath}))\right\|$$

$$I = \frac{1}{\mu_0}\left\|E_{max}B_{max}\sin^2\{kz-\omega t\}\left(\hat{\jmath}\times(-\hat{\imath})\right)\right\|$$

$$I = \frac{E_{max}B_{max}}{\mu_0}\sin^2\{kz-\omega t\}\left\|+\hat{k}\right\|$$

$$I = \frac{E_{max}B_{max}}{\mu_0}\sin^2\{kz-\omega t\}$$

$$I = \frac{E_{max}\left(\frac{E_{max}}{c}\right)}{\mu_0}\sin^2\{kz-\omega t\}$$

$$I = \frac{E_{max}^2}{c\mu_0}\sin^2\{kz-\omega t\}$$

Note: the time average of $\sin^2\{kz-\omega t\}$ is $\frac{1}{2}$…this is where the AVERAGE intensity formula gets its ½!!!

$$I = \left(13.07\,\frac{\text{mW}}{\text{m}^2}\right)\sin^2\left\{(1.48\times10^6\ \text{m}^{-1})z - \left(4.44\times10^{14}\,\frac{\text{rad}}{\text{s}}\right)t\right\}$$

f) In the real world, some atoms might absorb energy as the wave travels throughout the universe.
If absorption is present, we assume the intensity decreases as the wave travels.
We were told the wave travels through empty space…assume no absorption…assume intensity is constant.
Note: "free space" just means "space free of all matter".

33.11

a) Green with just a kiss of blue.
Random side note 1: this is a common solid state laser wavelength.
Random side note 2: the human eye is most sensitive to the nearby wavelength 555 nm.

b) Laser light diverges very slowly (can be treated as plane waves).
We are told to can assume the beam circular beam cross-section has uniform intensity.

$$I_{avg} = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}}{\pi r^2} = \frac{10.0 \times 10^{-3}\,\mathrm{W}}{\pi(3.00 \times 10^{-3}\ \mathrm{m})^2} = 35\underline{3}.7\,\frac{\mathrm{W}}{\mathrm{m}^2}$$

c) WATCH OUT! Electromagnetic radiation leaving the surface of the sun is radiating *spherically*.
The area is not *circular* but is instead *spherical*!
Using $\mathcal{P} = 3.90 \times\ 10^{26}$ W & $R_{sun} = 6.96 \times 10^8$ m

$$I_{avg} = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}}{4\pi r^2} = \frac{3.90 \times\ 10^{26}\ \mathrm{W}}{4\pi(6.96 \times 10^8\ \mathrm{m})^2} = 64.\underline{0}7\,\frac{\mathrm{MW}}{\mathrm{m}^2}$$

d) We know

$$I_{avg} = S_{avg} = \frac{E_{max}^2}{2c\mu_0} = \frac{cB_{max}^2}{2\mu_0}$$

From this we find

$$E_{max} = \sqrt{2c\mu_0 I_{avg}} = \sqrt{\frac{2c\mu_0\mathcal{P}}{\pi r^2}} = 51\underline{6}.4\,\frac{\mathrm{V}}{\mathrm{m}}$$

$$B_{max} = \sqrt{\frac{2\mu_0 I_{avg}}{c}} = \sqrt{\frac{2\mu_0\mathcal{P}}{\pi r^2 c}} = \frac{E_{max}}{c} = 1.7\underline{2}1\ \mu\mathrm{T}$$

Remember, these are the field *amplitudes*…not the fields (which vary in time and over space).

e) We know $k = \frac{2\pi}{\lambda} = 1.1\underline{8}1 \times 10^7\ \mathrm{m}^{-1}$ and $c = \frac{\omega}{k} \rightarrow \omega = kc = 3.5\underline{4}3 \times 10^{15}\,\frac{\mathrm{rad}}{\mathrm{s}}$.

f) We are told the wave propagates along the $+x$ axis and the electric field oscillates parallel to the $\hat{\jmath}$ axis.
Therefore

$$\vec{E}(t) = E_{max}\sin\{kx - \omega t\}\hat{\jmath}$$

$$\vec{E}(t) = \left(51\underline{6}.4\,\frac{\mathrm{V}}{\mathrm{m}}\right)\sin\left\{\left(1.1\underline{8}1 \times 10^7\ \mathrm{m}^{-1}\right)x - \left(3.5\underline{4}3 \times 10^{15}\,\frac{\mathrm{rad}}{\mathrm{s}}\right)t\right\}\hat{\jmath}$$

We know the propagation direction is determined by the pointing vector $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$ whose direction is

$$\hat{S} = \hat{E} \times \hat{B}$$

The first input to the cross-product is $\hat{E} = +\hat{\jmath}$ and the output is $\hat{S} = +\hat{\imath}$. This tells us $\hat{B} = +\hat{k}$.

$$\vec{B}(t) = B_{max}\sin\{kx - \omega t\}\hat{k}$$

$$\vec{B}(t) = \left(1.7\underline{2}1\ \mu\mathrm{T}\right)\sin\left\{\left(1.1\underline{8}1 \times 10^7\ \mathrm{m}^{-1}\right)x - \left(3.5\underline{4}3 \times 10^{15}\,\frac{\mathrm{rad}}{\mathrm{s}}\right)t\right\}\hat{k}$$

33.12

a) The photon energy is

$$E_\gamma = \frac{hc}{\lambda} \approx \frac{1240\ \text{eV}\cdot\text{nm}}{632.8\ \text{nm}} = 1.96\ \text{eV}$$

Alternatively, one could use

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.626\times 10^{-34}\ \text{J}\cdot\text{s})\cdot\left(3.00\times 10^{8}\ \frac{\text{m}}{\text{s}}\right)}{632.8\times 10^{-9}\ \text{m}} = 3.1\underline{4}1\times 10^{-19}\ \text{J} = 1.96\ \text{eV}$$

b) This problem is screaming out for unit analysis.
We know power which is energy over time.
We know energy per photon.
We want photons per sec.

$$E_\gamma\cdot\left(\frac{\text{Number of }\gamma'\text{s}}{\text{second}}\right)\quad \text{has units of }\ \frac{\text{J}}{\text{s}} = \text{W} \ldots \text{the units of } \mathit{power}$$

Therefore we expect

$$\frac{\text{Number of }\gamma'\text{s}}{\text{second}} = \frac{\mathcal{P}}{E_\gamma} = \frac{5.00\times 10^{-3}\ \frac{\text{J}}{\text{s}}}{3.1\underline{4}1\times 10^{-19}\ \text{J}} = 1.5\underline{9}2\times 10^{16}\ \frac{\text{photons}}{\text{second}}$$

c) The photon momentum (magnitude) is $p = \frac{E_\gamma}{c} = \frac{h}{\lambda} = 1.0\underline{4}7\times 10^{-27}\ \text{kg}\cdot\frac{\text{m}}{\text{s}}$.

d) Most people call it red or orangish-red.

33.13

One method is to throw the laser *away from the ship*. Since she and the laser are initially at rest the momentum equation becomes

$$\vec{p}_{i\ astronaut} + \vec{p}_{i\ laser} = \vec{p}_{f\ astronaut} + \vec{p}_{f\ laser}$$
$$0 = \vec{p}_{f\ astronaut} + \vec{p}_{f\ laser}$$
$$\vec{p}_{f\ astronaut} = -\vec{p}_{f\ laser}$$

The astronaut will move *towards* the ship if she throws the laser *away from* the ship.
Similarly, the astronaut can simply turn on the laser and point it away from her ship.
The photons traveling away from the ship carry momentum with them.
To conserve momentum, the astronaut must gain momentum the other direction.

$$\vec{p}_{i\ astronaut} = \vec{p}_{f\ astronaut} + \vec{p}_{f\ photons}$$
$$0 = \vec{p}_{f\ astronaut} + \vec{p}_{f\ photons}$$
$$\vec{p}_{f\ astronaut} = -\vec{p}_{f\ photons}$$

The only problem? Despite having so many photons per second, each photon momentum is miniscule.
It would take quite a while to gain speed this way.
Think: in the previous problem we had a $\mathcal{P} = 10.0$ mW laser producing $\frac{\#}{s} = 3.18\times 10^{16}\ \frac{\text{photons}}{\text{second}}$.
Each photon had momentum (magnitude) $p_\gamma = 1.047\times 10^{-27}\ \text{kg}\cdot\frac{\text{m}}{\text{s}}$.
Each second the laser is one the photons carry away momentum (magnitude)

$$p_{total\ per\ sec} = \left(\frac{\#}{\text{s}}\right)\cdot(1.00\ \text{s})\cdot p_\gamma \approx 3.33\times 10^{-11}\ \text{kg}\cdot\frac{\text{m}}{\text{s}}$$

If you had 1000 times the power and left it on for an entire day (86400 s $\approx 10^5$ s) you would bring a 1.0 kg mass up to a speed of about $3\,\frac{\text{mm}}{\text{s}}$. HA!
All this being said, the astronaut could in theory turn on the laser until it runs out of battery then throw it away to get as much possible momentum out of the laser.
I will have you do a problem numerically in a minute to verify you remember how to do this.

33.14

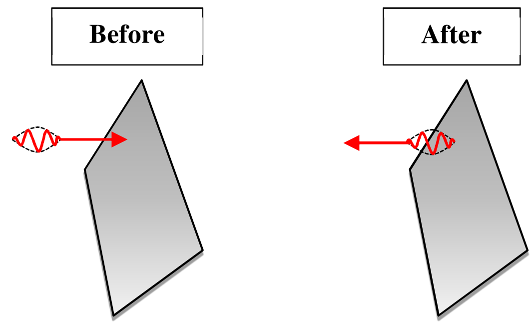
a) A giant piece of aluminum foil in space would be illuminated by light from the sun.
The photons from the sun would impact the foil.
The foil would reflect the photons (assumes foil is $\approx$ 100% reflective…only *approximately* true).
When the photons reverse direction, their momentum *magnitude* is unchanged but momentum *vector* does change!
The change in momentum of a single photon is

$$\Delta\vec{p}_\gamma = \vec{p}_f - \vec{p}_i$$
$$\Delta\vec{p}_\gamma = p_\gamma(-\hat{\imath}) - p_\gamma(+\hat{\imath})$$
$$\Delta\vec{p}_\gamma = -2p_\gamma\hat{\imath}$$

The *magnitude* of momentum *change* is thus

$$\|\Delta\vec{p}_\gamma\| = 2p_\gamma$$

The aluminum foil changed the momentum of the light.
By Newton's third law, the light changes the momentum of the aluminum foil (in the opposite direction).
The foil feels a force away from the sun due to this radiation.
We call this radiation pressure or radiation force.



b) Perhaps this was intuitive when you think, farther from the sun, light intensity should decrease. Radiation force on the sail should decrease. Acceleration will decrease as distance from the sun increases.

Since we have been discussing momentum and forces, an FBD seems wise (see FBD at right).

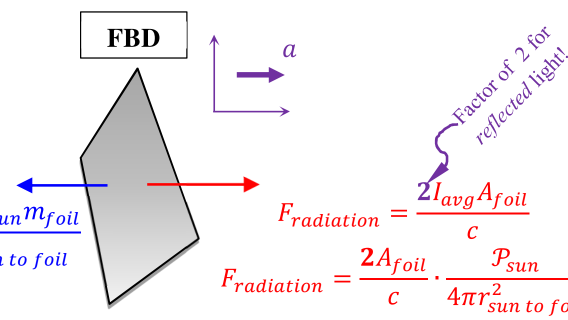
$$F_{radiation} - F_G = m_{foil}a$$

Note: I am assuming the foil is far enough from the earth and moon such that the sun is essentially the only significant source of gravitational attraction.

$$\frac{2A_{foil}}{c}\cdot\frac{\mathcal{P}_{sun}}{4\pi r^2_{sun\ to\ foil}} - \frac{GM_{sun}m_{foil}}{r^2_{sun\ to\ foil}} = m_{foil}a$$

$$a = \left(\frac{2A_{foil}\mathcal{P}_{sun}}{4\pi c m_{foil}} - GM_{sun}\right)\frac{1}{r^2_{sun\ to\ foil}}$$

Notice we expect the acceleration of the foil to decrease as it gets farther from the sun.



$$F_G = \frac{GM_{sun}m_{foil}}{r^2_{sun\ to\ foil}}$$

$$F_{radiation} = \frac{2I_{avg}A_{foil}}{c}$$

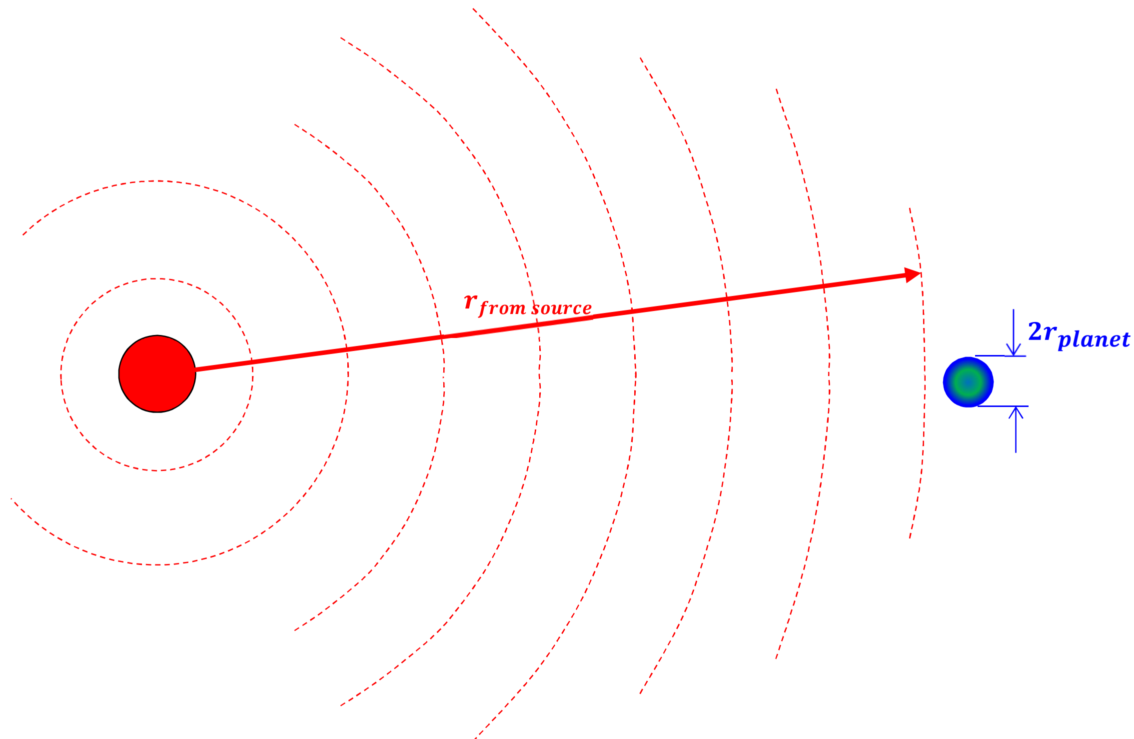
$$F_{radiation} = \frac{2A_{foil}}{c}\cdot\frac{\mathcal{P}_{sun}}{4\pi r^2_{sun\ to\ foil}}$$

Energy radiates spherically from the sun (in all directions equally). The power output of the sun over the *spherical* area $4\pi r^2$ describes the intensity $I_{avg} = \frac{\mathcal{P}_{sun}}{4\pi r^2}$ of electromagnetic radiation distance r from the sun.

c) When light *reflects* off the sail we get twice the radiation pressure on the sail (compared to a sail which *absorbs* light). We would expect a shiny sail to reflect more light than a black sail.

33.15 Parts a & b) In this problem we treat the radio source as a point source radiating energy *spherically*. By the time those *spherical* waveforms reach our planet, we treat them like *plane* waves. What gives? Before doing the problem, consider the figure below. The point source is on the left radiating spherically. Our planet is on the right, far from the source. The waves are still spreading out spherically throughout all of space. HOWEVER, at astronomically large distances from the source, even objects as large as planets have negligible size compared to the spherical wavefront radius. Notice: all points on the planet receive the signal at essentially the same angle. This is the essence of a plane wave.

PART C)



The intensity of the signal at the planet's location is

$$I_{from\ source\ at\ planet's\ position} = \frac{\mathcal{P}_{source}}{4\pi r_{from\ source}^2}$$

The power of the signal received by a circular dish is

$$\mathcal{P}_{received} = I_{from\ source\ at\ planet's\ position} \cdot \pi r_{dish}^2$$

$$\mathcal{P}_{received} = \frac{\pi r_{dish}^2}{4\pi r_{from\ source}^2} \mathcal{P}_{source}$$

$$\boldsymbol{\mathcal{P}_{received}} = \frac{\boldsymbol{A_{dish}}}{\boldsymbol{A_{wavefront}}} \boldsymbol{\mathcal{P}_{source}}$$

Notice the power received relates to source power using a ratio of dish area to spherical wavefront area.

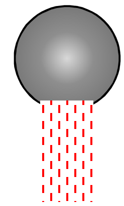
Note: 1 lightyear $= \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right) \times \left(365.25 \text{ days} \times \frac{24 \text{ hrs}}{1 \text{ day}} \times \frac{3600 \text{ s}}{1 \text{ hr}}\right) = 9.4\underline{6}7 \times 10^{15}$ m

$$\mathcal{P}_{source} = \frac{4r_{from\ source}^2}{r_{dish}^2} \mathcal{P}_{received}$$

$$\mathcal{P}_{source} = \frac{4\left(9.4\underline{6}7 \times 10^{15} \text{ m}\right)^2}{(50 \text{ m})^2}(1.00 \times 10^{-15} \text{ W}) = \mathbf{1.434 \times 10^{14}\ W}$$

33.16 DISCLAIMER: Problem is not very realistic but it does introduce the concept of optical levitation.

a) We are told the beam, directed upwards, is reflected downwards.
Side note: when a beam reverse direction like this, we say it is retro-reflected.
The radiation force (magnitude) for 100% retro-reflection is



$$F_{rad} = \frac{2I_{avg}A_{receiving}}{c}$$

$$F_{rad} = \frac{2\left(\frac{\mathcal{P}_{beam}}{A_{beam}}\right)A_{receiving}}{c}$$

In this case, we are told to assume the entire beam has uniform intensity and can be treated as a plane wave. Since the entire beam is incident upon the sphere (and because we are told to assume all the light gets retro-reflected) $A_{receiving} = A_{beam}$!

$$F_{rad} = \frac{2\mathcal{P}_{beam}}{c}$$

We expect this force must balance the force of gravity pulling down on the bead.
This implies the radiation force *magnitude* must equal mg.

$$\frac{2\mathcal{P}_{beam}}{c} = mg$$

$$\mathcal{P}_{beam} = \frac{mgc}{2}$$

b) To get density involved, recall $\rho = \frac{m}{V} \quad \rightarrow \quad m = \rho V$.

$$\mathcal{P}_{beam} = \frac{\rho V gc}{2}$$

$$\mathcal{P}_{beam} = \frac{\rho\left(\frac{4}{3}\pi R^3\right)gc}{2}$$

$$\mathcal{P}_{beam} = \frac{\rho\left(\frac{4}{3}\pi\left(\frac{D}{2}\right)^3\right)gc}{2}$$

$$\mathcal{P}_{beam} = \frac{\rho\left(\frac{\pi}{6}D^3\right)gc}{2}$$

$$\mathcal{P}_{beam} = \frac{\pi\rho gcD^3}{12}$$

$$\mathcal{P}_{beam} = \frac{\pi\left(2200\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)\left(3.00\times10^8\,\frac{\text{m}}{\text{s}}\right)(25\times10^{-6}\text{ m})^3}{12}$$

$$\boldsymbol{\mathcal{P}_{beam} \approx 26.5\text{ mW}}$$

Note: in real life, optical levitation is achieved using a *focused* laser beam.
This allows one to use a much lower laser power.

c) If we used a perfectly absorbing bead, the radiation force would cut in half.
We expect only half the bead's weight is canceled by the laser.
The acceleration would be downwards at half the rate of freefall ($a_y = -\frac{g}{2}$).

I think I can feel confident that you have done way more than enough work by now.
Don't do the rest of the problems.
That's said, if you actually do want solutions…ask me.

33.17 Not done yet.
33.18 Not done yet.
33.19 Not done yet.
33.20 Not done yet.
33.21 Not done yet.
33.22 Not done yet.