Physics Workbook Volume 4

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Use a laptop, tablet, or phone to access the solutions at my website above.

Find the chapter you want then open that link on your device.

The questions (this book) and the answers (on your device) ready to go at the same time!

Tip: you can search the solutions file for the problem number to skip directly to the solution you want (hit CTRL-F).

Chapter 21: Electric Forces

Learn how to correctly use the equation for Coulomb *force*.

- Distinguish between *charge* and *magnitude of charge*.
- Distinguish between *force* and *magnitude of force*.
- Note how symmetry can simplify computation.
- Use appropriate prefixes and conversion factors in numerical data.
- Plot force (or force *magnitude*) for a range of different cases.

Coulomb Force VECTOR 1	Coulomb Force VECTOR 2	Coulomb Force MAGNITUDE
$\vec{F}_{1\text{on}2} = \frac{kq_1q_2}{r_{1\text{to}2}^2}\hat{r}_{1\text{to}2}$	$\vec{F}_{1\text{on}2} = \frac{kq_1q_2}{\boldsymbol{r}_{\mathbf{1to2}}^{\mathbf{3}}}\vec{\boldsymbol{r}}_{\mathbf{1to2}}$ The r-*hat* changes to r-*vector* & distance *squared* changes to distance *cubed*.	$F_{12} = \frac{k\lvert q_1\rvert\lvert q_2\rvert}{r_{12}^2}$

$\vec{F}_{12}$ means the force of charge 1 <u>on</u> charge 2

F_{12} means the *magnitude* of the force between charges 1 and 2

$\vec{r}_{12}$ means the vector <u>from</u> the center of 1 <u>to</u> the center of 2

r_{12} means the distance (no direction) <u>from</u> 1 <u>to</u> 2

$\hat{r}_{1to2}$ means the direction only <u>from</u> 1 <u>to</u> 2…this is a unit vector

q_1 means charge 1…you should include the sign of the charge

$|q_1|$ means magnitude of charge 1…you should NOT include the sign of the charge

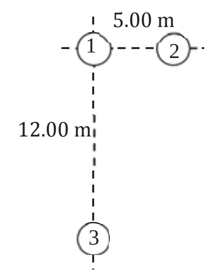
Coulomb constant	Permittivity of free space	Relationship
$k = k_e = 8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$	$\varepsilon_0 = 8.85 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}$	$k = \frac{1}{4\pi\varepsilon_0}$ $\quad\varepsilon_0 = \frac{1}{4\pi k}$
Proton mass	**Electron mass**	***MAGNITUDE* of electron charge**
$m_p = 1.673 \times 10^{-27}$ kg	$m_e = 9.109 \times 10^{-31}$ kg	$e = +1.602 \times 10^{-19}$ C

While these are just constants, I do have a few comments.

- I think of k_e as 9E9 (like "ninety nine"…while cheesy, maybe it helps that number stick)
- This (ε) and this (ϵ) are epsilon but this ($\mathcal{E}$) is capital script E while this (Ɛ) is the Euler constant. Notice the subtle differences. When doing homework online, mouse over the epsilon before clicking on it to ensure you are actually using epsilon.
- Obviously e has another meaning relating to natural logarithms. The context of a problem statement usually makes clear which e is to be used in a problem.
- It is worth restating that in E & M problems e is the ***<u>magnitude</u>*** of the charge on an electron.
    - The charge on a proton is $+e$.
    - The charge on an electron is $-e$.
    - The *magnitude* of electron charge is $+e$.
    - Remember how $g = +9.8\,\frac{\mathrm{m}}{\mathrm{s^2}}$ is the *magnitude* of the acceleration due to gravity? The acceleration due to gravity in freefall was $\vec{a} = -g\hat{\jmath}$.

21.1 Determine the units of the Coulomb constant k by looking at $F_{12} = \frac{k|q_1||q_2|}{r_{12}^2}$.

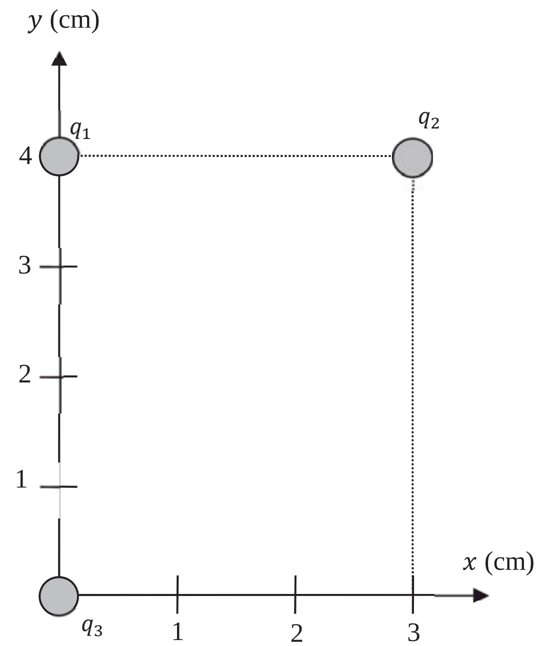
21.2 Three particles are shown at right. Particles 1 & 2 are 5.00 m apart while particles 1 & 3 are 12.00 m apart.



a) Determine the *displacement* vector $\vec{r}_{23}$ that points from 2 to 3.
b) Determine the *unit* vector that points from 2 to 3.
c) Determine the *unit* vector that points from 1 to 2.
d) Determine the *unit* vector that points from 1 to 3.
e) Determine the *unit* vector that points from 3 to 2.

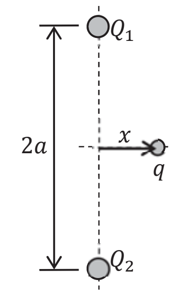
21.3 Assume $q_1 = +1.00$ μC, $q_2 = -2.00$ μC, and $q_3 = -3.00$ μC. For now, assume only electrical forces act on the charges.

a) Without using any math, sketch the estimated directions of $\vec{F}_{13}$ and $\vec{F}_{23}$ in the figure. Try to estimate which force arrow should be larger (or are they about the same size)?
b) Sketch the estimated direction of the <u>NET</u> force ON q_3? Hint: tail-to-tip, tail-to-tip.
c) Determine the net force (magnitude <u>and direction</u>) on q_3. Note: unless otherwise specified, when asked for a direction you are expected to determine *a numerical value* for an angle and include a small sketch labeling your angle for clarity.



21.4 Use a simulation to show how symmetry is used to estimate force direction and simplify force magnitude computations.

21.5 Welcome to the land of pain



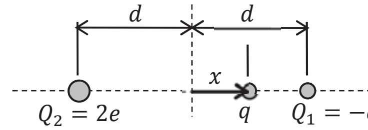
Two point charges are located at fixed positions on the y-axis (see upper figure at right). Assume $Q_1 = Q_2 = +e$. One charge is distance a above the origin while the other lies distance a below the origin. A third point charge, q, lies at position x on the x-axis. This charge is also positive with magnitude $|q| = e$. To be clear, we will want to consider both positive and negative values of x for this problem.

a) Think *before* computing. Which way should the net Coulomb force on q (due to the other two charges) point? Be sure to consider the direction of the net force for $x > 0$, $x < 0$, and $x = 0$.
b) Determine an algebraic expression for the net Coulomb force exerted on q by the other two charges. Check the units of your final answer. Also, compare your result to what you expected from part a.
c) What value of x gives a maximum force? What is the maximum value of the force?
d) Assuming $x \ll a$, use the binomial expansion to determine an approximate equation for the force. The answer should be of the form $F = slope \cdot x$.
e) For $x \gg a$, use the binomial expansion to determine an approximation for the force.
f) Plot the Coulomb force vs x assuming $a = 1.0$ nm and $e = +1.6 \times 10^{-19}$ C. Use values of x ranging from -5.0 nm up to $+5.0$ nm in 0.1 nm increments. Assume positive values of force indicate a force to the right while negative values of force indicate a force to the left.
   Think *after* computing.
   i. Do the signs on your plot match your intuition from part a for the directions of the force?
   ii. Does the force maximum at the appropriate position?
   iii. Can you get a numerical value for the slope close to the origin and compare it to part d.
   iv. You should also be able to plot the function from part e on the same graph. For large values of x we expect the function from part e and our function from part b to be nearly identical.
g) Think about what would happen to charge q if it was free to move along the x-axis and was released from rest. Be sure to consider all three cases: $x > 0$, $x < 0$, and $x = 0$.
h) **WHAT IF???** Suppose $Q_1 = Q_2 = +e$ while $q = -e$. Think through the entire problem again. Some parts might not change that much or at all. What would change? How would the plot change? How would your answers to part h change? Don't redo the work, just think then check my work in the solutions.
i) **WHAT IF???** Suppose $Q_1 = q = +e$ while $Q_2 = -e$. Think through the entire problem again. Some parts might not change that much or at all. What would change? This one is a little trickier to discuss…part h is wildly different, no? Don't re-compute; just think then check my work in the solutions.

Calculus based electricity and magnetism is no joke.

- You must understand concepts, figures, geometry, trig, calc, various notations, prefixes, and numerical computation.
- PLEASE take this class seriously and start practicing at least 2 hours a day for at least 5 days a week. By doing this you will have time to stay up on other classes during exam weeks.
- You must get a C or better to transfer. Why not play it safe and *over*-study for the first month? This is better than putting your career goals on hold.
- It is virtually impossible to completely solve *every* problem in this book. It is important for you to do some problems all the way through, then finish the rest by thinking only about how to set-up the problems.

21.6 Pain can be one dimensional: Three point charges are arranged as shown in the figure. Two charges are fixed in place. These charges are $Q_1 = -e$ and $Q_2 = +2e$. The charge ($q = -e$) can be located at any arbitrary position along the horizontal axis (left or right of the origin). To be clear, x is the horizontal *position* of q and x can be positive or negative while d is a *distance* (always positive).



a) Think *before* you compute.
   i. Which way will the force point for $-d < x < d$?
   ii. Which way will the force point for $x < -d$?
   iii. Which way will the force point for $x > d$?
   iv. Will there be 0, 1, 2, or 3 equilibrium points? Recall, an equilibrium point implies $\vec{F}_{NET\ on\ q} = 0$.
   v. Does your answer to part iv change your mind about part iii? What happens if $x \gg d$?

b) Determine the net Coulomb force acting on q for $x > d$.

c) Determine the net Coulomb force acting on q for $x < -d$.

d) Determine the net Coulomb force acting on q for $-d < x < d$.

e) At what location is q in equilibrium?

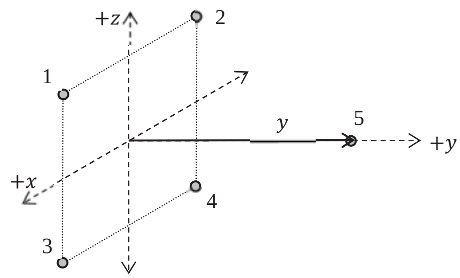
f) Assume $d = 1.0$ nm. Plot $\vec{F}_{NET\ on\ q}$ vs x for $-5d < x < +8d$ using increments of 0.1 nm. Note: the formula will not make sense if q is exactly on top of one of the other two charges (when $x = \pm d$)! You may have trouble making the plot. If you do have trouble plotting, delete the value of F for $x = \pm d$.

g) Think *after* you compute. Does your plot agree with your reasoning on part a?

h) **WHAT IF???** Suppose $q = +e$ instead of being a negative charge. Rethink the entire problem and consider what, if anything, would change.

i) **WHAT IF???** Suppose you kept $q = -e$ but Q_2 charge was changed to $Q_2 = -2e$. Rethink the entire problem and consider what, if anything, would change.

21.7 Pain can be 3D: Four identical positive charges (with magnitude q) are arranged on a square of side s that lies in the xz-plane as shown in the figure. A fifth identical point charge is located at position $(0, y, 0)$ from the origin on the y-axis.



a) Determine the Coulomb force of 1 on 5. It will look *ugly*.

b) What direction is the *net* Coulomb force on 5? Hint: symmetry.

c) Determine the *net* force on 5. Isn't symmetry awesome?!?!?

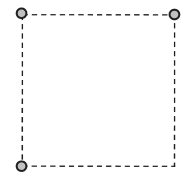
d) Think: How would the problem change if the fifth charge was negative instead of positive?

e) Think: How would things change if charges 1 and 2 were negative while charges 3, 4, and 5 were positive? Still assume all charges have equal *magnitude* q.

f) Think: How would things change if charges 1 and 3 were negative while charges 2, 4, and 5 were positive? Still assume all charges have equal *magnitude* q.

g) Now make some plots. Start by assuming all charges are positive with the same magnitude as an electron. Assume one side of the square is $s = 1.0$ nm.

21.8 Consider a square of sides s charges on three of the corners. The top left corner has charge $-q$ while the other two corners each have charge $+q$.



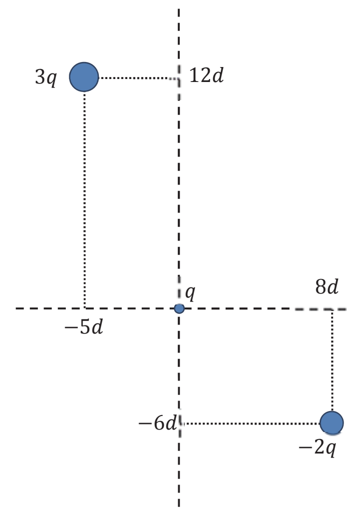
a) Determine the net force (magnitude and direction) on a fourth charge ($+q$) placed at the fourth corner.

b) Suppose, instead of placing the fourth charge at the fourth corner, the fourth charge is placed at the *center* of the square. Determine the net force, magnitude and direction.

c) Is there an equilibrium position for the fourth charge? Is there more than one? Is there no equilibrium position? If yes, where is it (or them)? If no, explain heuristically why no equilibrium position exists.

Note: if you are a clever person, on the last part you might think to re-align the figure such that the main diagonal of the square becomes the x-axis. If you don't see this clever trick, don't worry; things are messier but still doable.

21.9 Three point charges are arranged as shown in the figure. The distance are noted as multiples of the arbitrary distance d. You may assume d has units of meters.

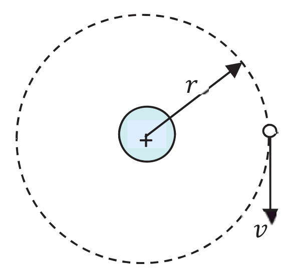


a) Determine the *magnitude* of the *net* electric force exerted on the mass at the origin. <u>Answer with a 3 digit decimal times</u> $\frac{kq^2}{d^2}$.

b) Determine the *direction* of the *net* electric force exerted on the mass at the origin. <u>Express your final answer in degrees with three sig figs.</u> Include a sketch showing the angle to help clarify your answer.

Note: when someone tells you to determine a direction, you are usually expected <u>to get a number for an angle</u> and draw a sketch unless otherwise specified.

21.10 Now consider an electron in a circular orbit around a proton (a crude model of the hydrogen atom). The proton and electron are typically separated by about 5.29×10^{-11} m. The mass of a proton is 1.67×10^{-27} kg while the mass of an electron is 9.11×10^{-31} kg. The <u>magnitude</u> of the charge (on both the proton and electron) is given as $e = 1.6 \times 10^{-19}$ C.



a) Determine the magnitude of the Coulomb force between the electron & the proton.

b) Determine the gravitational force acting on the electron.
Assuming it is near the earth's surface you can just use $m_e g$.

c) Take the ratio of the weight to the electrical force.
Do you think it is reasonable to ignore gravitational forces for this type of problem?

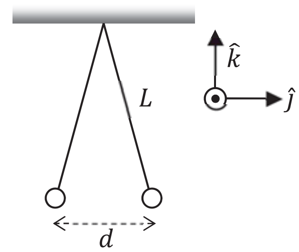
d) Determine the velocity of the electron in its orbit. You may assume constant speed for this crude model.

21.11 Pretend for a moment that gravity does not exist. Suppose the moon is held in circular orbit about the earth using a Coulomb force (instead of gravity). Assume the earth and moon each carry the same magnitude of charge uniformly distributed. Assume the earth has mass 5.97×10^{24} kg and radius 6.37×10^{6} m. The moon has mass 7.35×10^{22} kg and radius 1.74×10^{6} m. The center-to-center distance from earth to moon is 3.84×10^{8} m. Assume the moon orbit requires 27.3 days.

a) What magnitude of charge would be required to keep the moon in orbit using a Coulomb force?

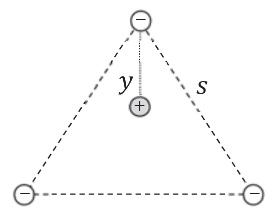
b) If we assume the moon is negatively charged, how many electrons are added to the moon?

21.12 Two balls, each mass m, hang from the ceiling using light strings. Each ball has static charge with unknown magnitude q. As a result, the two balls repel from one another and hang at the angle shown in the picture. The distance of separation is d and the length of each string is L. Assume the size of each ball is negligible compared to the length of the string. Assume the origin of the coordinate system is the point where the two strings attach to the ceiling. To be clear, the figure shows the equilibrium state.



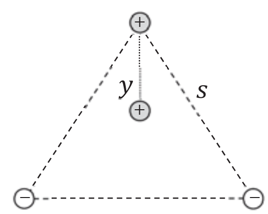
a) Determine the angle between each string and the vertical in terms of L and d. This angle is called the *half-angle* as it is half of the angle between the two strings.
b) Determine the magnitude of static charge on each ball. Hint: FBD.
c) Are the balls both positive, both negative, or one of each?
d) Suppose one ball had more charge than the other. Would the balls hang at different angles from the vertical or would one ball swing out more than the other?

21.13 Three equal *negative* charges $-q$ are arranged at the corners of an equilateral triangle of side s. A fourth charge $+q$ is placed distance y from the top charge.



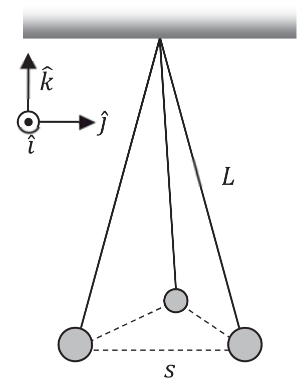
a) Is there some value for y such that the 4th charge is in equilibrium?
b) What value of y is required for the fourth charge to be halfway between the bottom two corners of the triangle?
c) What is the net force on the 4th charge when the 4th charge is halfway between the bottom two corners?
d) Write an equation for the net force on the fourth charge for arbitrary position y *inside the triangle*. Do not worry about above the triangle ($y < 0$) or below the triangle ($y > \frac{\sqrt{3}}{2}s$).

21.14 A positive charge $+q$ is placed at the top corner of the equilateral triangle with negative charges (each $-q$) placed at the bottom corners of the triangle. A fourth charge, positive with *unknown* magnitude Q, is placed distance y from the from the top. Assume $Q \neq q$.



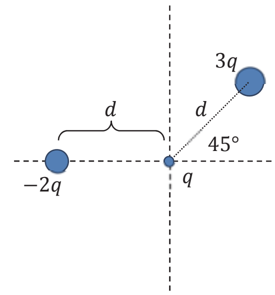
a) Without doing any math, do you think there should be an equilibrium point? If so, would it be inside or outside of the triangle? Is there more than one possibility for an equilibrium point?
b) **GNARLY…AVOID:** As a particularly brutal challenge, determine the equilibrium point (or points). The next part is much better practice for the test for typical undergrad courses…do that instead.
c) **Really good practice!** You are told the net force *magnitude* on the 4th charge is F when it is located at the centroid. Determine an expression for Q in terms of k, q, F, and s.

21.15 This time *three* balls (each mass m) are hanging from strings. Each ball again has static charge with unknown magnitude q. The balls separate from each other and form an equilateral triangle in the horizontal plane. The side of the equilateral triangle is s and the length of each string is L. Assume the size of each ball is negligible compared to the length of the string. Assume the origin of the coordinate system is the point where the three strings attach to the ceiling. To be clear, the figure shows the equilibrium state.



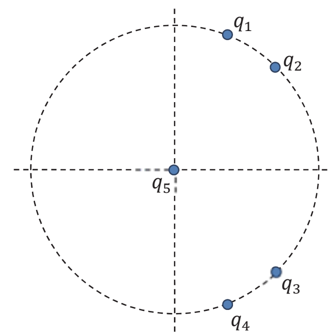
a) Determine the distance between the triangle's center & any of the balls in terms of s.
b) Determine the distance between triangle's center & the origin in terms of L & s.
c) At what angle, from the vertical axis, does each ball hang in terms of L & s?
d) Determine the magnitude of charge on each ball.
e) If the balls each had slightly different charges, would the problem still be symmetric? Would the spheres still lie in an equilateral triangle?

21.16 Consider three charges placed as shown in the figure.



a) Where should you place a fourth charge ($q_4 = +4q$) to put charge q at the origin into equilibrium? Give your answer as an (x, y) coordinate where x & y are each written in terms of d.
b) Where should you place the 4th charge if it had the same magnitude but was negative?
c) What if? Suppose we wanted the 4th charge to be exactly distance d from q. What new value of q_4 would need to be used? Would the angle also need to change?

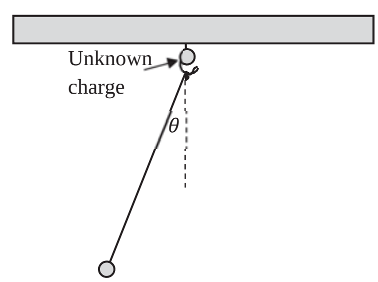
21.17 Assume four charges lie on a circular arc of radius R. A fifth charge is at the origin. All charges have the same *magnitude* but different signs. Use symmetry to determine the *direction* of the net force on the charge at the origin for each case below.



a) First assume all are positive.
b) What if q_1 & q_4 are negative and the rest are positive?
c) What if q_2, q_3 & q_5 are negative and the rest are positive?
d) What if q_1 & q_2 are negative and the rest are positive?
e) What if q_1, q_3 & q_5 are negative and the rest are positive?

The point of this problem is to emphasize how symmetry considerations can dramatically simplify problems.

21.18 A simple pendulum is made from a string of length L and a small ball of mass m and charge magnitude q. At the point where the string attaches to the ceiling a second point charge (unknown magnitude and sign) is located. The system is free to swing normally; the upper point charge doesn't add friction at the pivot or anything like that. In this problem the ball is always released from rest at an angle θ from the vertical.



a) Will the presence of the charges change the speed of the ball at the bottom of the swing? Explain why (or why not).
b) After doing several experiments, student learn the maximum tension *without* the unknown charge present is T_{max}. When the unknown charge *is* present T_{max} is *reduced* by 10%. Are the charges the same sign or opposite signs?
c) Determine the unknown charge magnitude.