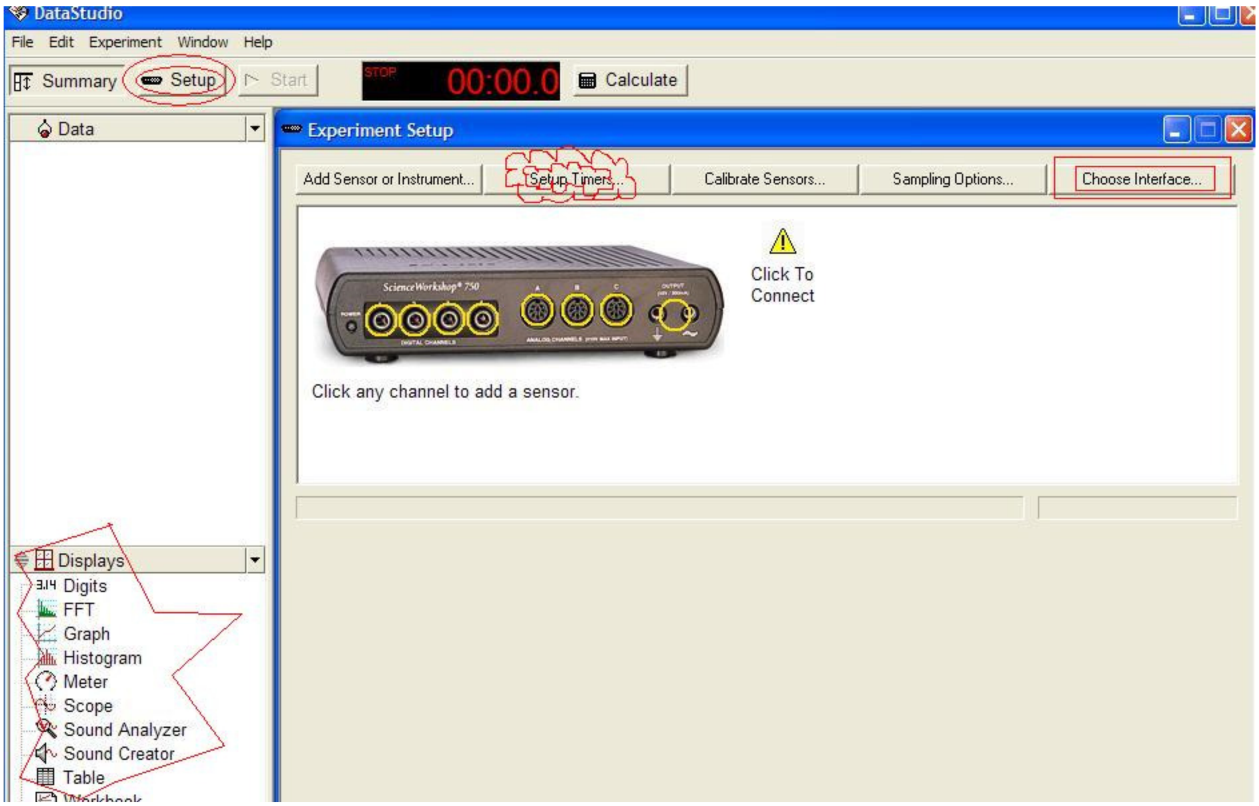
# Physics Lab Manual Appendices

Revised Summer 2015

## Appendix A: DataStudio Setup

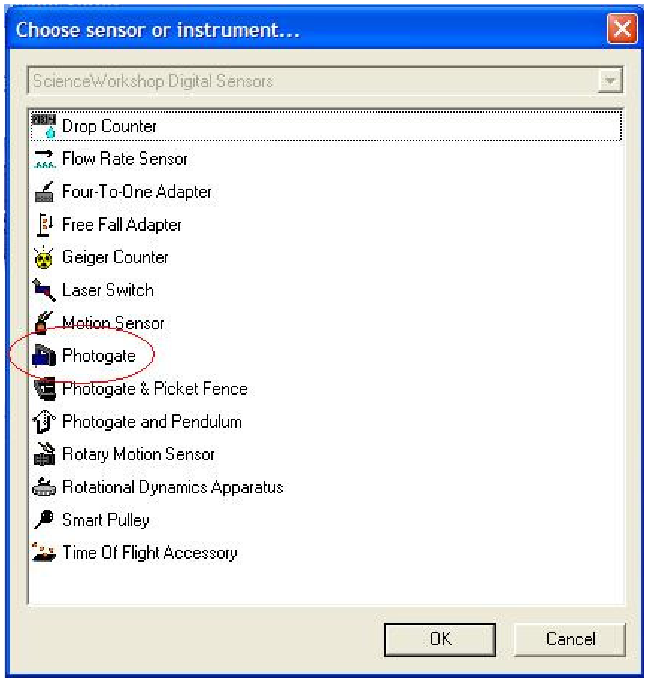
1) Make sure the Pasco interface is turned on by checking the green light.
2) Make sure the USB cable is connected to the computer.
3) Open DataStudio.
4) Click CREATE EXPERIMENT. You can cancel the pop-up window at the bottom of the screen.
5) In the top left corner of the screen you will find the buttons SUMMARY, SETUP, and START. Click on the SETUP button. See the figure below (it is circled).
6) A pop-up window (*Experiment Setup*) appears and has five buttons in it. The right most button says CHOOSE INTERFACE. Click the CHOOSE INTERFACE button. See the figure below (it is in a box).
7) A new pop-up opens. Select *ScienceWorkshop 750* and click OK. You should now see a picture of the interface box in the *Experiment Setup* window with yellow circles on it. See figure below (it should look like that).



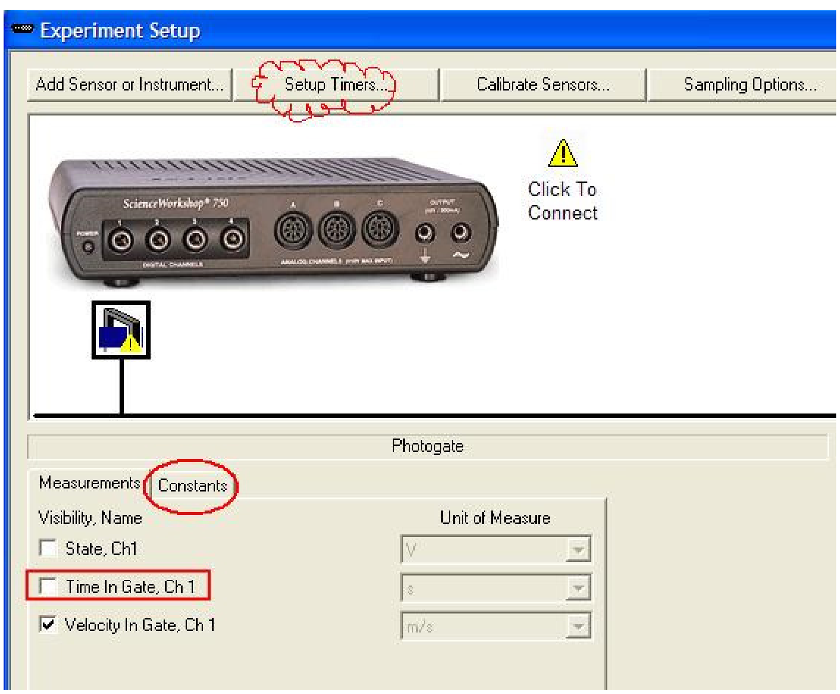

8) Before clicking on a circle, connect the sensor or sensors you will need to either the Digital Channels or Analog Channels of the interface box. The Digital Channels are the four plug-ins on the left while the Analog Channels are the three plug-ins in the middle. The far left plugs can be used as a power supply (AC or DC).
9) For each sensor you connect, click on the appropriate location on the computer screen. Select the appropriate sensor for each channel in the pop-up box that appears. See below for specifics on Photogates.
10) Lastly, click the START button on the top toolbar (in the top left corner next to the SETUP button) when you are ready to start taking data.

## Appendix B: DataStudio Photogates

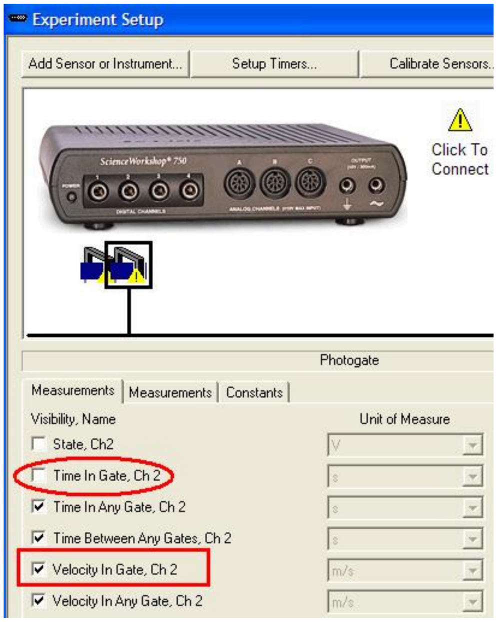
1) After clicking on the channel you'll see the box below pop-up. Find the photogate (see circled), click on it, and click ok to select it.



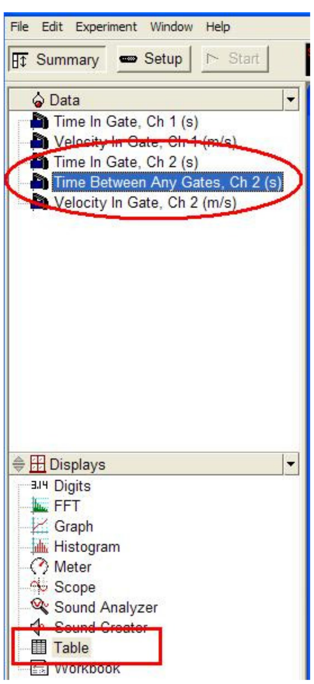

2) Once you click ok the screen will look like the one below. Click on the "Time in Gate, Ch 1" button. Click on the CONSTANTS tab you can enter the length of the black flag attached to the top of the glider. It should be about 10 cm but you can get an extra sig fig by actually measuring it with a ruler (or the ruler on the air track). Now the computer will automatically divide the length of the flag by the time in the gate which gives you the average velocity of the flag (and thus the glider) at the position of the gate.

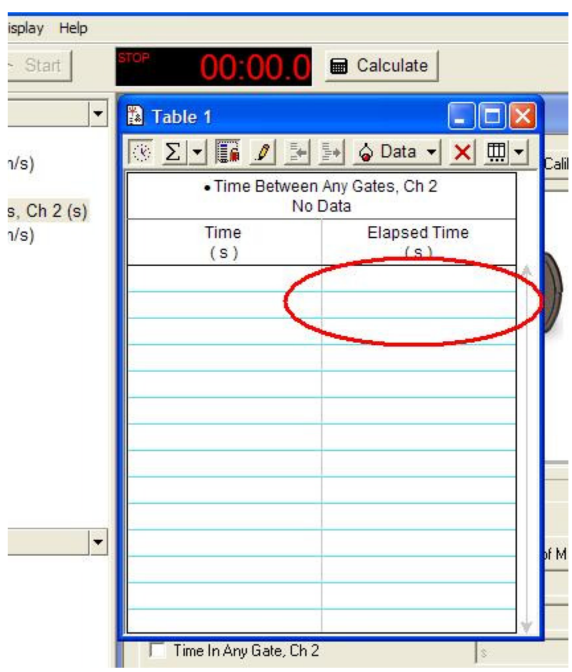


3) IF TWO PHOTGATES ARE USED, other options become available (see figure below). Any options you don't need you can get rid of by un-clicking the appropriate box. Some options you might want are "Time in Gate, Ch 2", "Velocity in Gate, Ch 2," or "Time Between any Gates, Ch 2". To select these just be sure the appropriate box has a check in it.

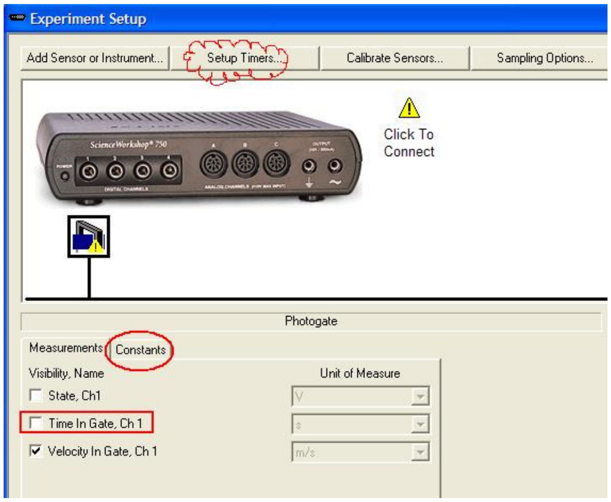



4) To actually record the measurements, you must tell the DataStudio program to display your measurements. to do this you must drag the measurement from the top left side of the screen (see circled in left figure) to the type of display you want at the bottom left side (see boxed in left figure). To read out a single number you could use DIGITS (say for a pressure reading or temperature reading). To see a graph you could use GRAPH (for the conservation of angular momentum lab). Typically it is easiest to use TABLE (times and velocities). Once you do this the screen will look light the right figure below. Once you hit the start button your data for the time between the gates would appear in the Elapsed Time column (circled in the right figure).

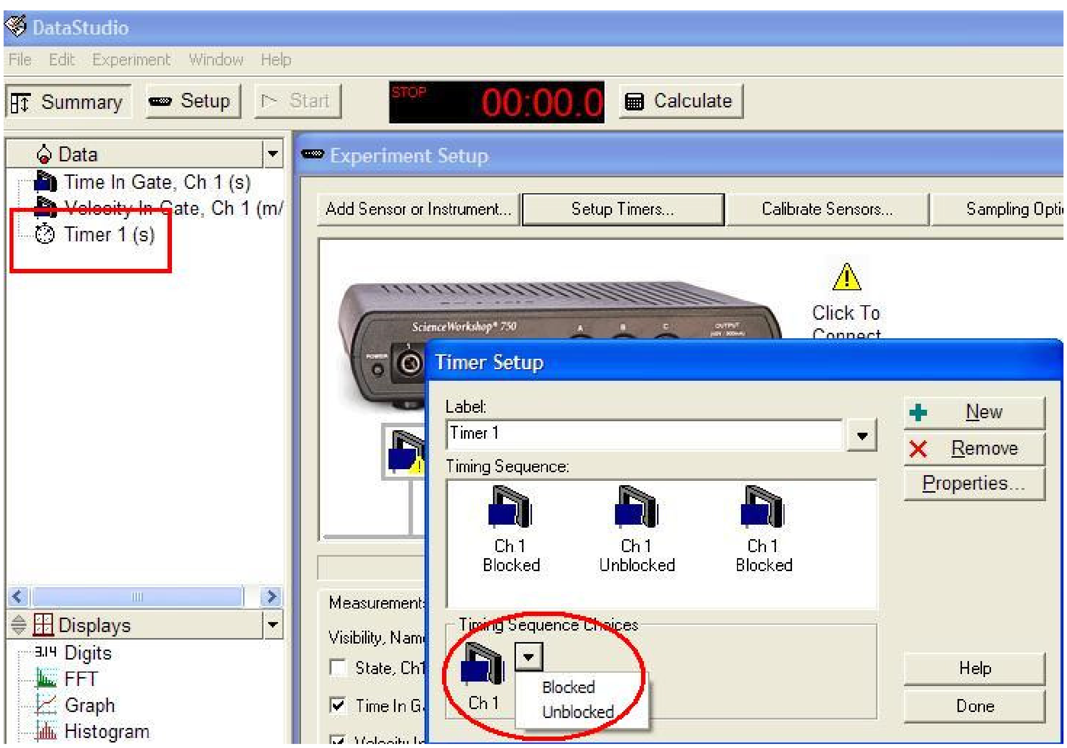


5) One other mode of timing exists and it requires you to setup a timer. This is used for timing oscillations. When a glider oscillates back and forth on the air track you can time it with a single photogate. When you set the glider oscillating you will notice that one complete oscillation occurs between the time the photogate is first temporarily blocked and the time when it is blocked again. This means you need a timer to record the time that a single photogate is BLOCKED, then UNBLOCKED, and lastly BLOCKED again. To do this click the SETUP TIMERS button (just above the picture of the interface).

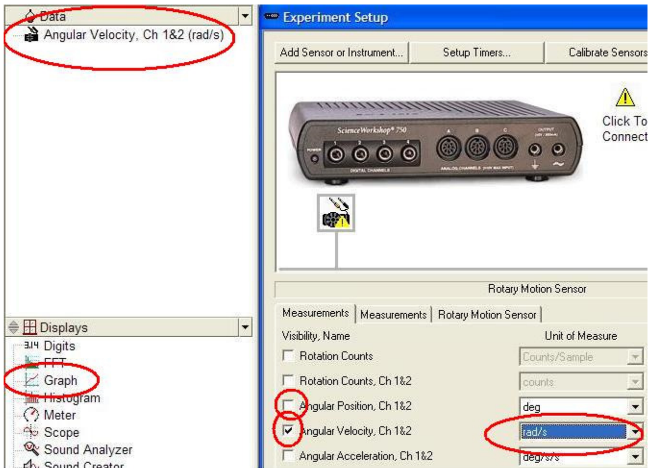



6) A pop-up box will appear. To setup the timer properly you will want it to read as shown. First click the arrow (circled) and select the BLOCKED option. Click the arrow again and select the UNBLOCKED option. Click the arrow a third time and select BBLOCKED again. Now click on done and "Timer 1" should appear in the top left side of the screen (see the boxed item in the figure). You can drag this "timer 1" down to table to see the measurements just as in the previous example.

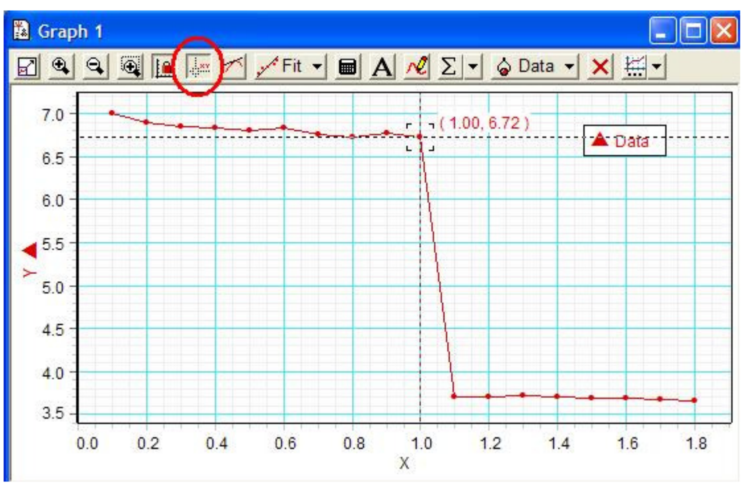


## Appendix C: DataStudio Rotary Motion Sensors

You can set up the Rotary Motion Sensor to read angular velocity (just like setting up a photogate). Verify you have the yellow cable from the Pasco "Rotary Motion Sensor" in digital input 1 on the interface box. The black cable goes into digital input 2. You can unclick the Angular Position button and click on the Angular Velocity button. **DONT FORGET TO CHANGE THE UNITS to rad/s!** Then drag the angular velocity (on top left side of figure) down to the graph (bottom left side of figure).

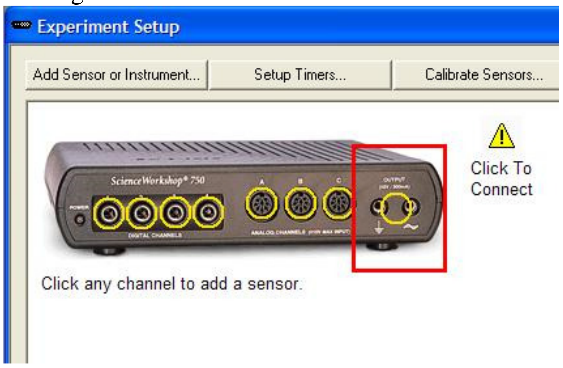



Spin the disk and hit the start button. Verify the computer is taking data. You should see a graph of angular speed versus time being plotted by the computer. It will look a little like the graph below. Think: if your graph is upside down the values are all negative. How could you fix the negative sign? How does the negative sign relate to the spinning? Try using the "xy tool", the 6$^{th}$ button from the left in the graph tool bar (see circled). When you click this button the dotted lines will appear. You can use these dotted lines to quickly determine the xy coordinates of any point. Simply bring the crosshairs over the point and the xy tool should lock onto the point and read out the coordinates to you. Notice that my graph left out the units…you are expected to figure them out.

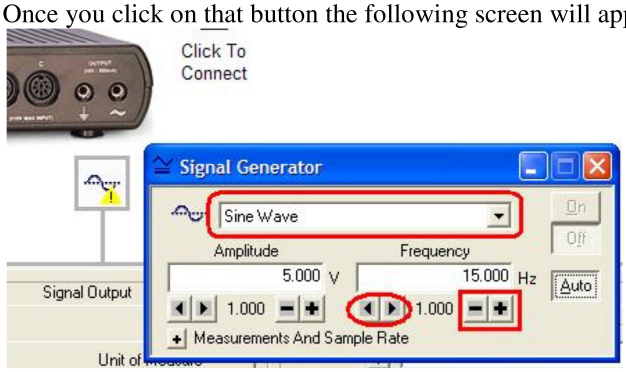


## Appendix D: DataStudio Power Output

Instead of clicking on the Digital Channels to connect a photogate, click on the Output channels on the far right of the diagram.




Once you click on that button the following screen will appear.

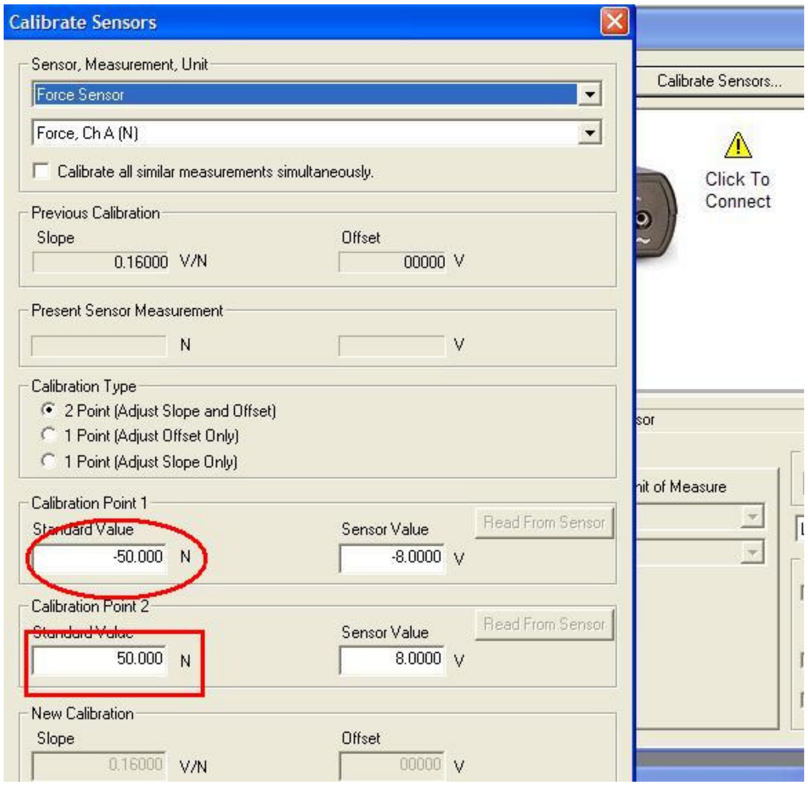



The drop-down arrow next to the "Sine Wave" text (rounded box) allows you to choose various types of power supplies. Most commonly used are Sine Wave or (if you scroll up after clicking on the arrow) "DC Voltage". The frequency can be changed by typing in a new frequency and hitting enter. Another way to change the frequency is to click the right and left arrows (circled) until you get to a convenient increment (say 1 Hz or 0.1 Hz). The frequency can then be increased or decreased (by the chosen increment) by hitting the + or – buttons (boxed). The Amplitude can be similarly adjusted.

BE AWARE OF THE CURRENT LIMIT. I believe it is 300 mA. If you use a small resistor, say 10 Ω, with the 5V DC output the current limit will be enabled such that no more than 3V is across the resistor. This can lead to student confusion.

## Appendix E: DataStudio Force Sensor Calibration

Once you have your force sensor running, click on the Calibrate Sensors button above the picture of the interface box in the Experiment Setup window.
Hang a 50 g mass on the sensor. This will be calibration point 1, standard value of 0.49 N.
Hang a 100 g mass on the sensor. This will be calibration point 2, standard value of 0.98 N.
Click Done.



## Appendix F: Report Guidelines

### GENERAL LAB WRITING TIPS

Each informal lab report you turn in should include all of these things (unless otherwise specified by your instructors). This list is by no means complete but these are spots that I often grade on.

- 1st page of report, upper right hand corner should say:
  Author: Billy Smith
  Partners: Susie Jones
  Rex Rexburg
- Neatness is important and as such you will lose points if your reports are considered to be sloppy.
- When writing reports by hand, use engineering paper and triple space things so you have room to make corrections without starting over from scratch.
- When first writing reports, start each section on its own piece of paper. This allows you to always keep your sections in the proper order. Reports with sections out of order may have their score reduced.
- Keep sentences short. Avoid run-ons. It is better to have two short sentences than one long one in most cases. It is ok if it reads in a boring way since this is scientific writing, not literature.
- Labs are written in <u>third person, past tense</u>. Exception: introduction/theory section can include present tense. Exception: some instructors allow first person in the conclusions for certain cases (see below).
  - Use the third person in the past tense (the object or it is typically the subject of the sentence).
    - "The group equated $a$ with $g$ and found blah blah blah." is an example in third person past tense.
  - No statements are commands in second person
    - "Equate $a$ with $g$ to …" is an example of a command in the second person (it is the same thing as saying "*You* equate $a$ with $g$ to …").
  - No statements are in first person
    - "I (or we) equated $a$ with $g$ to …" is an example of a sentence in first person
    - <u>Only in your conclusion section</u> may you use first person statements but they should be avoided unless they dramatically simplify the sentence structure.
- Avoid using words that end in "ing".
- Diagrams are to be a ½ page in size.

## INTRODUCTION TIPS

- Look at the end of chapter review sections in your textbook. These show excellent styles for writing equations.
- Think: what equation was the starting point for the lab? What were the derived results you used for comparing your prediction to the experiment? Both the starting point equations and the final result equation should be in your theory. Leave out the crap in between.
- Look at your data sheet. Any variables appearing in your data sheet should be probably be mentioned in your introduction at first appearance. Do not redefine the variables at second use. For the remainder of the lab only use the variable in your writing to save time (the reason for introducing a variable in the first place). Below is an example; notice the following:
    - The variables used in the intro match those in the table
    - The first time the variable is used it is defined (typically just before or after the variable is used in an equation)
    - <u>The definitions of the variables are in paragraph form.</u> **<u>The variables are NOT defined in a bulleted list like this:</u>**
        - $g$=the magnitude of the acceleration due to gravity
        - $t$=elapsed time
    - When the variable is used a second time the definition is not needed (only the variable is used)
    - Third person, present tense, passive voice is used
    - A sentence that ends just after an equation needs no period (look in your book for this precedent)
    - Never include %difference or error equations in the introduction (only in the calculations section)
    - Notice a table is captioned ABOVE the table (while a figure is captioned BELOW the figure).

**Introduction:**

This lab determines if Ohm's law is valid for a light bulb. Ohm's law for is written

$$\Delta V_d = I_d R \tag{1}$$

where $\Delta V_d$ is the voltage across an ohmic device, $I_d$ is the current through the device, and $R$ is the resistance of the device. Solving equation (1) for $R$ gives

$$R = \frac{\Delta V_d}{I_d} \tag{2}$$

A linear plot of $\Delta V_d$ versus $I_d$ shows that a device is indeed ohmic.

**Data:**

Table 1 – Current versus voltage data for the test device.

| $\Delta V_d$ (V) | $I_d$ (A) |
|---|---|
| 1.00 | 4.0 |
| 0.75 | 3.0 |
| 0.50 | 2.0 |

# MORE INTRODUCTION TIPS

- In your introduction, it is often confusing what equations to include. Start with key equations out of the book/theory. Ignore minor intermediate steps and finish with your final derived equation that was made from your theory equations. Look at the following example and notice:
  - The first time the variable is used it is defined (this one was interesting because *F* was defined in the first paragraph but used in an equation for the first time in the second paragraph).
  - When the variable is used a second time the definition is not needed (only the variable is used).
  - A sentence that ends just after an equation needs no period (look in your book for this precedent).
  - Kinematics and Newton's second law are key concepts that make for a logical starting point for your desired audience (someone with basic physics knowledge).
  - All the intermediate steps of the algebra are left out (the derivation is then shown in the Calculations section).
  - Third person, present tense is used

**Introduction:**

Kinematics states that vertical displacement ($\Delta y$) of an object dropped from rest is given by

$$\Delta y = \frac{1}{2}gt^2 \tag{1}$$

where $g$ is the magnitude of acceleration due to gravity, and $t$ is elapsed time. Furthermore, Newton's second law states that the force $F$ acting on a falling mass is given by

$$F = ma \tag{2}$$

where $m$ is a falling mass and $a$ is the acceleration of the falling mass.

By equating $a$ with $g$ and combining equations (1) and (2) the ratio of displacement to force is

$$\frac{\Delta y}{F} = \frac{t^2}{2m} \tag{3}$$

**Calculations:**

Derivation of $\frac{\Delta y}{F}$.

$$\Delta y = \frac{1}{2}gt^2$$

$$\frac{2\Delta y}{t^2} = g$$

$$F = ma$$

$$F = mg \text{ since } a=g$$

$$F = m\frac{2\Delta y}{t^2}$$

$$\frac{F}{\Delta y} = \frac{2m}{t^2}$$

## PROCEDURE AND DIAGRAM TIPS

- The procedure uses third person, past tense. Write in paragraph form (not in bulleted lists).
  - This example is sufficiently detailed so that another student could reproduce the experiment.
  - It refers to the apparatus diagram (Figure 1) so it is easier to understand.
  - The equation $v=D/t$ is a simple enough equation that it can be put in line with the text.
  - Numbers with two digits or more are written as numbers ("10" not "ten").
  - Numbers with only one digit are written as words ("five" not "5").
  - In instances where a more than one number appears all numbers are written in numerals not words ("3 and 12" not "three and 12").
  - The figure is about ½ a page in size.

**Procedure:**

A ball of diameter $D$ was dropped from the known height $y$ (see Figure 1). The laser switch was positioned a distance $h$ below the release point. The ball passed through the laser switch operating at 200.0 Hz. The DataStudio program determined $v$ from the time $t$ the ball blocked the switch and $D$ using the approximation $v=D/t$.

The students measured $v$ for ten values of $h$. At each $h$, five velocities were averaged to get $v_{avg}$. A plot of $v_{avg}^2$ versus $h$ was made to verify that the square of the speed increased linearly with $h$.

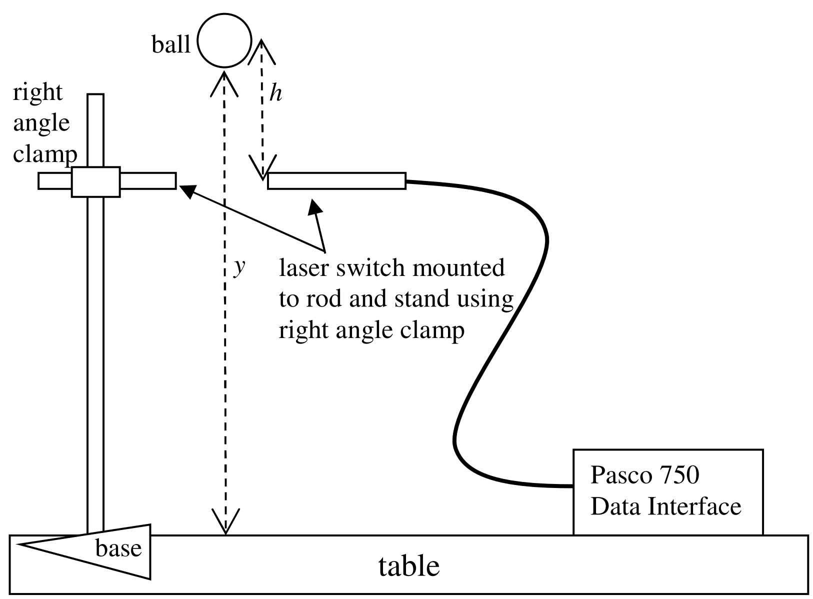



Figure 1: A laser switch measured $v$ of a ball a distance $h$ below the release point.

## DATA & CALCULATIONS TIPS

- Never alter the sig figs of recorded numbers. Suppose you record three values with some measuring device. The scale gives you 1.503 g·s/cm$^3$, 90.0 g·s/cm$^3$, and 123.45 g·s/cm$^3$. You would record these numbers similar to the table shown below:
    - Notice that the units are listed with the variable (not on the number).
    - The variable is in italics while the units are not in italics.
    - Even though Excel tried to eliminate the trailing zero for the number 90.0 I manually corrected it to clearly show that I measured the mass with error in the tenths column (not the ones column).
    - Not all numbers will have the same number of sig figs.
    - The symbols ^ and * are not appropriate. Use superscript or subscript where appropriate. The "middle dot" symbol can be found typically by clicking INSERT-SYMBOL and then searching in the pop-up window for the middle dot symbol.

| $b_1$(g·s/cm$^3$) | $b_2$(g·s/cm$^3$) | $b_3$(g·s/cm$^3$) |
|---|---|---|
| 1.503 | 90.0 | 123.45 |

- %errors or %precisions have no units and should have only one sig fig
    - Exception: when the %error or %precision exceeds 10%, use two sig figs.
    - Exception: when the first digit of an error or percent is 1, use two sig figs.
- Your calculation section should include all your free-body diagrams and sketches used to understand equations OTHER THAN APPARATUS DIAGRAMS.
- Derivations and sample calculations belong in the calculations section.
    - Be sure the variables in your derivations match the variables in your data table.
    - An example of a derivation that includes sufficient detail was seen in the MORE INTRODUCTIONS TIPS section.
- For each major calculation (excluding %difference) show a sample. This sample shows the algebraic equation, a step with the numbers put in, then the final answer with units and appropriate sig figs. See the example below:
    - For several equations with identical algebra show only one sample calculation. For example, for the entire data table below, the only sample calculation needed is:

$$a = \frac{2y}{t^2} = \frac{2(2.00)}{0.88^2} = 4.7\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

    - Notice the unusual data only has 2 sig figs for the time in the first 2 values of *a* (thus the calculated value of *a* has only 2 sig figs)
    - Notice the third calculation of has 3 sig figs because both *x* and *t* have 3 sig figs in that calculation.

| $x$ (m) | $t$ (m) | $a$ (m/s$^2$) |
|---|---|---|
| 1.00 | 0.65 | 4.7 |
| 2.00 | 0.88 | 4.7 |
| 3.00 | 1.78 | 4.98 |

- Any numeric values in your report should have units and appropriate levels of sig figs (see the above example).

## MORE DATA & CALCULATIONS TIPS

- When calculations are performed on your data, you must think about the sig figs of calculated values.
  - If a column of data uses an average and standard deviation, the standard deviation is rounded to one significant digit. The data above it is rounded to match the column of the standard deviation. See the example below, where $v_i$ is calculated from $x$ and $y$ (using some formula with $g$ in it).
    - Notice that you would expect the calculation to have 3 sig figs (both $x$ and $y$ have 3 sig figs) but since the data has such a large spread (large stdev) the calculation only makes sense to 2 digits (in this case the tenths column).
    - Notice that the standard deviation is not 0.1 (or some other number you type in) but rather a computed value.

| trial | $x$ (m) | $y$ (m) | $v_i$ (m/s) |
|---|---|---|---|
| 1 | 3.21 | 1.06 | 6.9 |
| 2 | 3.33 | 1.06 | 7.2 |
| 3 | 3.09 | 1.06 | 6.6 |
| 4 | 3.28 | 1.06 | 7.1 |
| 5 | 3.20 | 1.06 | 6.9 |
| | | avg | 6.9 |
| | | stdev | 0.2 |

  - If for some reason the standard deviation is vanishingly small, use sig figs rules or propagation of error (see the appendices of lab manual) to determine the sig figs for your calculated values.
    - Notice that sig fig rules imply that if $x$ and $y$ have 3 sig figs then the calculated value $v_i$ should not have MORE than 3 sig figs. Even thought the stdev indicates the data is good to the thousandths column we know that data cannot be known beyond the hundredths column due to the sig figs of this particular example.

| trial | $x$ (m) | $y$ (m) | $v_i$ (m/s) |
|---|---|---|---|
| 1 | 3.21 | 9.16 | 2.35 |
| 2 | 3.21 | 9.16 | 2.35 |
| 3 | 3.20 | 9.16 | 2.34 |
| 4 | 3.21 | 9.16 | 2.35 |
| 5 | 3.20 | 9.16 | 2.34 |
| | | avg | 2.34 |
| | | stdev | 0.004 |

The table below shows what data you calculated, whether or not a sample calculation would be expected, and the reasoning behind inclusion (or exclusion) of a sample calculation.

| Data Calculated | Include Sample Calculation? | Reasoning |
|---|---|---|
| $\mu$ calculated ten times | only show ONE! | verify you can do this type of calculation |
| $\mu_{avg}$ | sometimes, usually not | verify you are doing things correctly |
| $\sigma_\mu$ (the standard deviation of $\mu$) | sometimes, usually not | verify you are doing things correctly |
| %precision | almost always show this | it is important to know how you estimated the errors |
| %difference | sometimes, usually not | verify you are doing things correctly |
| in general: anything in the Excel sheet you simply record | never | no calculation to check |
| in general: anything in the Excel sheet you had to compute (like a mass or a distance) | always | need to verify your computation by hand |
| avg, stdev, and %diff | at instructor's discretion | after a while this is redundant |

# MORE DATA & CALCULATIONS TIPS

**WHEN SHOWING CALCULATIONS BY HAND DO THESE STEPS**

1) Solve the equation algebraically
2) Plug in the numbers without doing any work
3) Show a few steps as needed
4) Show your final unrounded answer
5) Show your final rounded answer with appropriate units

**Example:** The underbar on certain digits (e.g. $1.0\underline{0}$) is used to track sig figs after each step.

$$\mu = \frac{m_2 g - (m_1 + m_2)\frac{2h}{t^2}}{m_1 g}$$

$$\mu = \frac{\left(0.40\underline{0}\right)\cdot(9.\underline{8}) - \left(0.25\underline{1} + 0.40\underline{0}\right)\cdot\frac{2(1.0\underline{0})}{(0.6\underline{5})^2}}{\left(0.25\underline{1}\right)\cdot(9.\underline{8})}$$

$$\mu = \frac{3.\underline{9}2 - \left(0.65\underline{1}\right)\cdot 4.\underline{7}33}{2.\underline{4}60} = \frac{3.\underline{9}2 - 3.\underline{0}82}{2.\underline{4}60} = \frac{0.\underline{8}38}{2.\underline{4}60} = 0.\underline{3}41 = 0.\underline{3}$$

This is an interesting example for several reasons:

1) Notice that the subtraction causes a loss of sig figs! Any time you subtract two numbers that are close together you want to watch out for the loss of a sig fig. Remember that in subtraction the LEAST SIGNIFICANT COLUMN is used to determine sig figs (not the NUMBER of sig figs).
2) In this unusual case the final answer has no units because the calculated quantity, $\mu$, is unitless. In general your final answers should have units.
3) Notice that extra digits are kept throughout the calculation to avoid intermediate rounding error. Usually one extra digit is plenty but I just wrote whatever I felt like out of the calculator (as long as it had at least one extra digit).
4) The two numbers with the least number of sig figs in this calculation are $t$=0.65s and $g$=9.8m/s$^2$. Percentage-wise, the error in $g$ is approximately 0.1/9.8≈1%. The error in $t$ is perhaps 0.1/0.65≈15% (this includes reaction time issues after synchronization). Assuming the error in $\mu$ is mainly attributable to these two measurements we would conservatively expect 20%error. Using this method of estimating the error gives $\mu = 0.341 \pm 20\% = 0.341 \pm 0.0\underline{6}8 = 0.3\underline{4} \pm 0.0\underline{7}$ which matches well with the number of sig figs obtained from rounding the standard deviation to one sig fig in the data table. This ignored the fact that $t$ was squared and the effects of the subtraction (both of which tended to increase errors). It also ignored the fact that $\mu$ was averaged (which tended to decrease errors). Notice that $0.3\underline{4} \pm 0.0\underline{7}$ is in good agreement with the value $0.\underline{3} \pm 0.\underline{1}$ obtained using sig fig rules show above.

## CONCLUSIONS TIPS

- In some cases (and with some instructors), first person is allowed for the conclusion. Only use first person if it dramatically simplifies the sentence structure. If in doubt, use third person.
- For informal reports, answer the conclusion questions in a numbered list.
- For formal reports, DO NOT answer the conclusion questions in a numbered list.

- When answering conclusion questions do not simply write a "Yes" or "No" answer. Support your answers with the reasoning behind your answer. Also, answer in a complete sentences that paraphrase the question.
    - If asked "Was the experimental result found to be in good agreement in today's lab?":
        - Best Answer: The experimental result $v_{exp}$= 6.2 m/s was in good agreement with $v_{th}$ =6.4 m/s because the percent precision was 4% while the percent difference was -3%.
            - Clearly states which theoretical value, in this case the velocity, was tested.
            - Supports claim by stating numerical results with %precision and %difference.
        - Bad answer (doesn't provide any support to your claim): The theory was proved true.
        - Worst Answer: Yes.

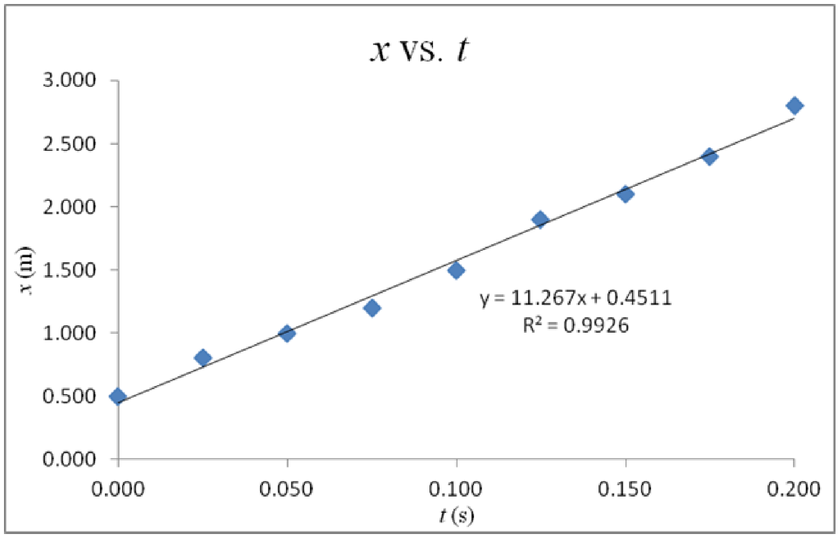
- When discussing errors in formal lab reports OR when asked a conclusion question on errors consider the following example:
    - In your conclusion you know that reaction time was a big factor in causing errors OR a question asks you:
      "Your reaction time introduces error in the experiment. Will your reaction time
      tend to make your %differences more positive or more negative?"
    - Suppose the formulas used in the lab are $F=ma$ and $y=1/2at^2$. These determined the theoretical result of

$$F_{th} = m\frac{2y}{t^2}$$

    - You think…hmm, my reaction time probably leads me to measure values of the time that are longer than they really were in the experiment.
    - This longer time is in the denominator of my theory value so it tends to make my theory value smaller.
    - The %difference is given by $\frac{exp-th}{th} \times 100\%$.
    - Because the theory value is smaller I would expect to be subtracting a smaller number (because of my reaction time) and thus have MORE POSITIVE (or LESS NEGATIVE) %differences.

## Appendix G: Type I Graph

| *t* (s) | *x* (m) |
|---|---|
| 0.000 | 0.500 |
| 0.025 | 0.800 |
| 0.050 | 1.000 |
| 0.075 | 1.200 |
| 0.100 | 1.500 |
| 0.125 | 1.900 |
| 0.150 | 2.100 |
| 0.175 | 2.400 |
| 0.200 | 2.800 |

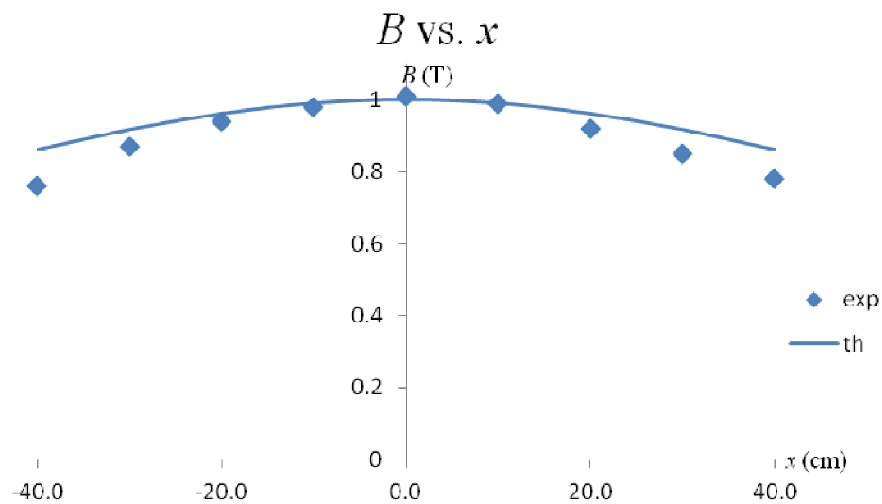



**Sample graph type 1:** The simplest type of plot has one data set with a small number of points.

1) Points are used for the data. The points are not connected by any line. Highlight your two columns of data in a single swoop (only select the numbers and don't use tricks with the shift or control keys), click the INSERT tab, then click the scatter plot button, and finally select the option that shows only points. <u>For a large number of points</u> use a smooth line with no points.
2) No gridlines used. Click on a gridline and hit the delete key.
3) A legend is not useful for a single set of data; delete the legend.
4) Always label each axis with units! Up at the top, Click on the Chart Tools: Layout tab.
5) In a formal report you would not use a title but would instead use a caption below. Since this graph was designed to go into an informal lab report a title was placed on the graph. This is also under the Chart Tools: Layout tab.
6) Variables are italicized, units are NOT italicized. I removed the bold. You can figure this out.
7) The data points take up a significant portion of the graph area; the points aren't bunched up in the top, bottom, or a corner of the graph. This is corrected by right clicking on the numbers to the side of the axis and formatting the axis. I changed this graph so that the *t*-axis only went out to 0.20 s instead of the 0.25 s Excel chose for me.
8) For this data set the theory states that *x* depends linearly on *t* (e.g. a constant speed problem). In this case, it was appropriate to add a LINEAR trendline. The $R^2$ value is shown as well as the equation of the trendline; the intercept is rarely set to zero. Ask what the $R^2$ means if curious.

## Appendix H: Type II Graph

| $x$ (m) | $B_{exp}$ (T) | $B_{th}$ (T) |
|---|---|---|
| -40.0 | 0.76 | 0.86 |
| -30.0 | 0.87 | 0.92 |
| -20.0 | 0.94 | 0.96 |
| -10.0 | 0.98 | 0.99 |
| 0.0 | 1.01 | 1.00 |
| 10.0 | 0.99 | 0.99 |
| 20.0 | 0.92 | 0.96 |
| 30.0 | 0.85 | 0.92 |
| 40.0 | 0.78 | 0.86 |




**Sample graph type 2:** This graph compares a data set of theoretically calculated values to experimentally obtained values.

1) Highlight your THREE columns of data in a single swoop (only select the numbers and don't use tricks with the shift or control keys), click the INSERT tab, then click the scatter button, and finally select the option that shows only points.
2) No gridlines used. Click on a gridline and hit the delete key.
3) A legend IS useful for multiple sets of data. Right click on the legend, hit Select Data, click Edit (you should be on Series 1), and type "exp" for the series name to indicate that data is the experimental data. Click ok and get returned to the first pop-up window. Before closing that pop-up, select Series 2, click Edit and change the name to "th" to indicate theory values. I believe Dom prefers using text boxes to distinguish between "exp" and "th".
4) Right click on any of the theoretical data points and select Format Data Series. Under Marker Options select NONE, under Line Color select solid line, under Line Style select a 1.5 point line and click the box that says "smoothed line" at the bottom of the pop-up.
5) Always label each axis with units! Up at the top, Click on the Chart Tools: Layout tab.
6) In a formal report use a caption below the graph (not a title), in an informal report you can use the title on the graph.
7) Variables in italics, units NOT italicized. I took off the bold as well. You can figure this out.
8) This is a unique case in terms of data spacing. Here all the values are grouped in the top third of the graph. I could've reduced the vertical axis range from 0.6 to 1.0. This was potentially misleading in my opinion. The variation in the magnetic field ($B$) would seem larger than it really was on that scale. I did, however, adjust the horizontal range to -40 to 40 since no data was taken beyond that range.

## Appendix I: Sample Report

**NOTE: This lab sample lab report is designed to look like your handwritten (informal) reports and not a formal report. Any formal report written must use the FORMAL REPORT GUIDELINES obtained elsewhere. That said, most aspects of handwritten reports are similar to formal reports. Some great resources for all things related to the communication of scientific information are the AHC Engineering Program Student Laboratory Handbook and http://writing.engr.psu.edu/index.html.**

Date: 12-21-11

Author: Billy Joe Smith

Partners: Reggie Smith, Patti Smythe

# Determination of the coefficient of friction between copper and steel using Fletcher's trolley

**Introduction:** This lab uses Newton's 2nd law to determine the coefficient of kinetic friction between copper and steel. Newton's 2nd law states that

$$\Sigma\vec{F} = m\vec{a} \tag{1}$$

where $\Sigma\vec{F}$ is the sum of forces (or net force) acting on an object, $m$ is the mass of the object, and $\vec{a}$ is the acceleration of the object. Both the net force and acceleration are vector quantities.

In this Fletcher's trolley experiment, a copper block ($m_1$) slides across a level steel surface as it is pulled by a hanging mass ($m_2$). The two masses are connected by a relatively massless string over a relatively massless and frictionless pulley (see Procedure section, Figure 1). Free body diagrams (see Calculations section, Figure 3) of the two masses result in the following force equations for $m_1$ and $m_2$:

$$\text{Block 1 } \Sigma F_x: \quad T - f = m_1 a \tag{2}$$

$$\text{Block 1 } \Sigma F_y: \quad n - m_1 g = 0 \tag{3}$$

$$\text{Block 2 } \Sigma F_x: \quad m_2 g - T = m_2 a \tag{4}$$

Here $T$ is the tension in the string connecting the two masses, $f$ is the frictional force between $m_1$ and the level surface, $a$ is the magnitude of the acceleration of $m_1$ in the $x$-direction, $n$ is the normal force exerted by the level surface on $m_1$, and $g$ is the magnitude of the acceleration due to gravity given by 9.8 m/s$^2$. Also note that here the subscripts $x$ and $y$ indicate the $x$ and $y$ directions of the coordinate system shown with the free body diagrams. In this case, a rotated coordinate system has been used for $m_2$ so that both blocks have the same $a$.

When two surfaces are in relative motion (such as $m_1$ sliding along the stationary level surface), $f$ is expressed as

$$f = \mu n \tag{5}$$

where $\mu$ is the coefficient of kinetic friction. Combining the above equations and solving for $\mu$ gives

$$\mu = \frac{m_2 g - (m_1 + m_2)a}{m_1 g} \tag{6}$$

An object moving with constant acceleration obeys the equation

$$\Delta x = \frac{1}{2}at^2 + v_0 t \tag{7}$$

where $\Delta x$ is the displacement of the object, $t$ is the elapsed time of motion, and $v_0$ is the initial velocity. For an object initially at rest $v_0 = 0$. For an object that travels a distance $h$ in the positive direction (as indicated by the coordinate system), $\Delta x = h$. Solving Equation (7) for $a$ gives

$$a = \frac{2h}{t^2} \tag{8}$$

Combining Equations (6) and (8) gives

$$\mu = \frac{m_2 g - (m_1 + m_2)\frac{2h}{t^2}}{m_1 g} \tag{9}$$

The accepted value for the coefficient of kinetic friction is $\mu_{acc}$ =0.36 from www.engineeringtoolbox.com/friction-coefficients-d_778.html.

**Procedure:** A steel plate was clamped to a level table and checked with a bubble level (see Figure 1). A massless, frictionless pulley was clamped at the opposite end of the table. The masses of the copper block $m_1$ and the hanging mass $m_2$ were determined with an electronic balance. A suitably large $m_2$ was chosen to ensure $m_1$ started to slide. A light string was attached to both $m_1$ and $m_2$. The string was long enough such that $m_1$ was still on the steel plate when $m_2$ was impacting the ground (see Figure 2).

The distance $h$ from the bottom of $m_2$ to the ground was measured with a meter stick. Then $m_2$ was released from rest. The time $t$ for $m_2$ to fall the distance $h$ was recorded with a stop-watch. The dropping of $m_2$ and the starting of the timer were synchronized by a count to reduce the error associated with reaction time. Equation (9) was used to determine the experimental values of $\mu$. The procedure was repeated ten times. The average of the ten trials was the experimental value $\mu_{exp}$.

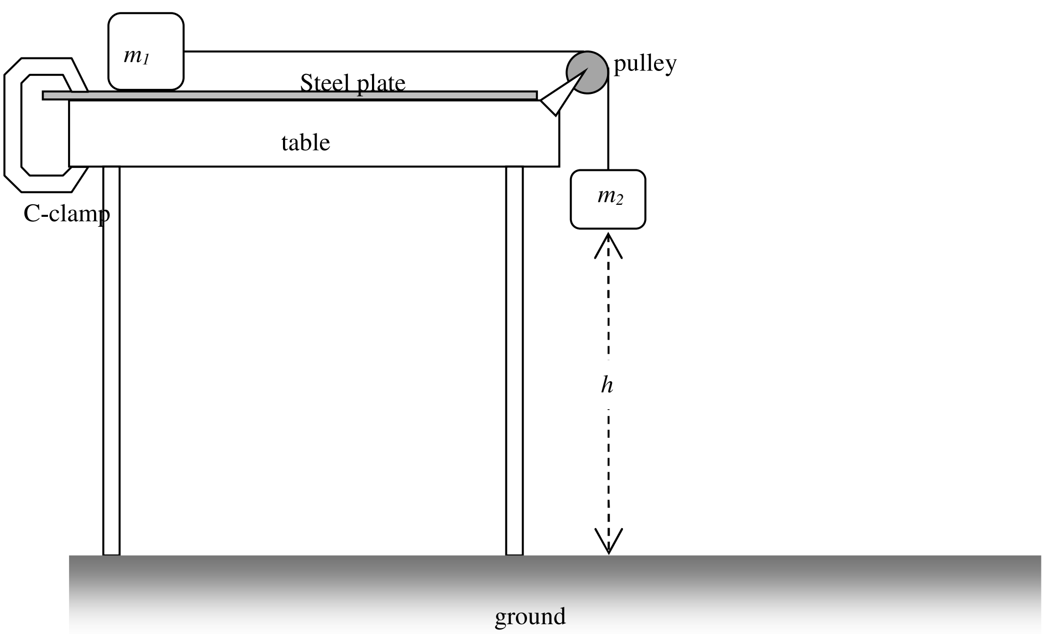



Figure 1 – Hanging mass $m_2$ is released from rest and falls a distance $h$ to the ground. A light, inextensible string connects $m_2$ to the copper block $m_1$ over a pulley that is essentially massless and frictionless (the dark circle in the figure). The flat steel plate is clamped to the table to prevent it from sliding along with $m_1$.

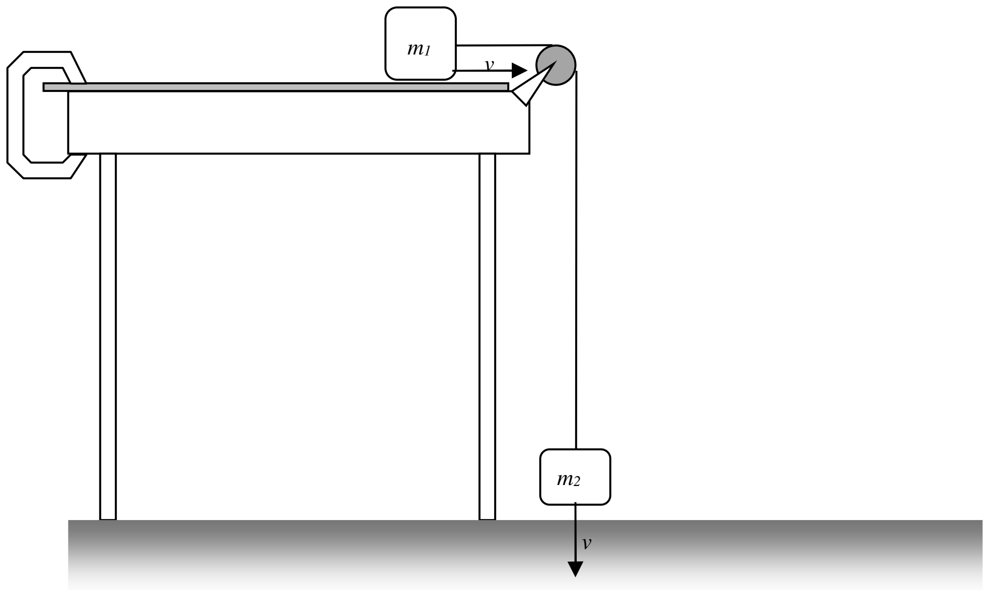



Figure 2 – After falling a distance $h$ both $m_1$ and $m_2$ are moving with the same speed $v$; both have traveled a distance $h$ and have the same acceleration $a$.

**Data:**

Table 1 – Time data for Fletcher's trolley using copper block on steel surface.

| $g$ (m/s$^2$) | $h$ (m) | $\mu_{acc}$ | $m_1$ (kg) | $m_2$ (kg) |
|---|---|---|---|---|
| 9.8 | 1.00 | 0.36 | 0.251 | 0.400 |

| Trial | $t$ (s) | $\mu$ |
|---|---|---|
| 1 | 0.65 | 0.34 |
| 2 | 0.66 | 0.38 |
| 3 | 0.65 | 0.34 |
| 4 | 0.64 | 0.30 |
| 5 | 0.67 | 0.41 |
| 6 | 0.66 | 0.38 |
| 7 | 0.66 | 0.38 |
| 8 | 0.63 | 0.26 |
| 9 | 0.65 | 0.34 |
| 10 | 0.65 | 0.34 |
| | avg | 0.35 |
| | stdev | 0.04 |

| %prec | %diff |
|---|---|
| 13 | -3 |

Error Estimate used
%prec = (stdev/avg)×100%

**Calculations:**

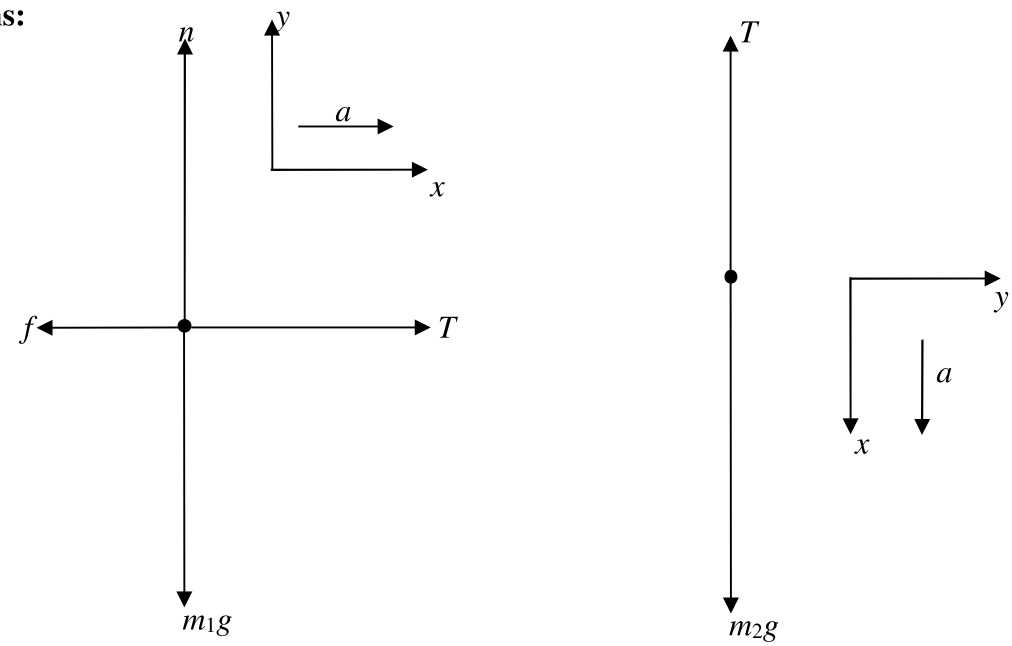



Figure 3 – The left diagram is the free body diagram (FBD) of $m_1$ while the right diagram is the FBD of $m_2$. Note the rotated coordinate system on $m_2$.

Block 1 $\Sigma F_x$: $T - f = m_1 a$ Block 2 $\Sigma F_x$: $m_2 g - T = m_2 a$

Block 1 $\Sigma F_y$: $n - m_1 g = 0$

$$\therefore n = m_1 g$$

$$f = \mu n = \mu m_1 g$$

$$T - \mu m_1 g = m_1 a$$

$$+ \qquad m_2 g - T = m_2 a$$

$$m_2 g - \mu m_1 g = m_1 a + m_2 a$$

$$m_2 g - (m_1 a + m_2 a) = \mu m_1 g$$

$$m_2 g - (m_1 + m_2) a = \mu m_1 g$$

$$\mu = \frac{m_2 g - (m_1 + m_2) a}{m_1 g}$$

$$\Delta x = \frac{1}{2} a t^2 + v_0 t$$

here $v_0 = 0$ and $\Delta x = h$ so

$$h = \frac{1}{2} a t^2$$

$$a = \frac{2h}{t^2}$$

$$\mu = \frac{m_2 g - (m_1 + m_2) \frac{2h}{t^2}}{m_1 g}$$

**In your report you would check the units…in this case the final answer should be unitless!*

Sample calculation of μ (uses first trial):

$$\mu = \frac{m_2 g - (m_1 + m_2)\frac{2h}{t^2}}{m_1 g}$$

$$\mu = \frac{\left(0.40\underline{0}\right) \cdot (9.\underline{8}) - \left(0.25\underline{1} + 0.40\underline{0}\right) \cdot \frac{2(1.0\underline{0})}{(0.6\underline{5})^2}}{\left(0.25\underline{1}\right) \cdot (9.\underline{8})}$$

$$\mu = \frac{3.\underline{9}2 - \left(0.65\underline{1}\right) \cdot 4.\underline{7}33}{2.\underline{4}60} = \frac{3.\underline{9}2 - 3.\underline{0}82}{2.\underline{4}60} = \frac{0.\underline{8}38}{2.\underline{4}60} = 0.\underline{3}41 = 0.\underline{3}$$

This is an interesting example for several reasons:

- Notice that the subtraction causes a loss of sig figs!
- The two numbers with the least number of sig figs in this calculation are $t$=0.65s and $g$=9.8m/s$^2$. Percentage-wise, the error in $g$ is approximately 0.1/9.8≈1%. The error in $t$ is perhaps 0.1/0.65≈15% (this includes reaction time issues after synchronization). Assuming the error in $\mu$ is mainly attributable to these two measurements we would conservatively expect 20%error. Using this method of estimating the error gives $\mu = 0.341 \pm 20\% = 0.341 \pm 0.0\underline{6}8 = 0.3\underline{4} \pm 0.0\underline{7}$ which matches well with the number of sig figs obtained from rounding the standard deviation to one sig fig in the data table. This ignored the fact that $t$ was squared and the effects of the subtraction (both of which tended to increase errors). It also ignored the fact that $\mu$ was averaged (which tended to decrease errors). Notice that $0.3\underline{4} \pm 0.0\underline{7}$ is in good agreement with the value $0.\underline{3} \pm 0.\underline{1}$ obtained using sig fig rules show above.

Sample calculation of %precision (using the estimate of stdev/avg):

$$\%precision = \frac{stdev}{avg} \times 100\% = \frac{0.04}{0.35} \times 100\% = 11.4\% \approx 10\%$$

- Notice that this differs in the second sig fig from the result in the data table. This is because Excel keeps sig figs beyond what is shown in the cell for all calculations. Said another way, my method shown here has intermediate rounding error while the method used in the data table (using Excel formulas) does not have intermediate rounding error. When rounding to one sig fig the answers are in good agreement as we would expect for an error calculation.

Sample calculation of %difference (using the estimate of stdev/avg):

$$\%difference = \frac{exp - th}{th} \times 100\% = \frac{0.35 - 0.36}{0.36} \times 100\% = -2.8\% \approx -3\%$$

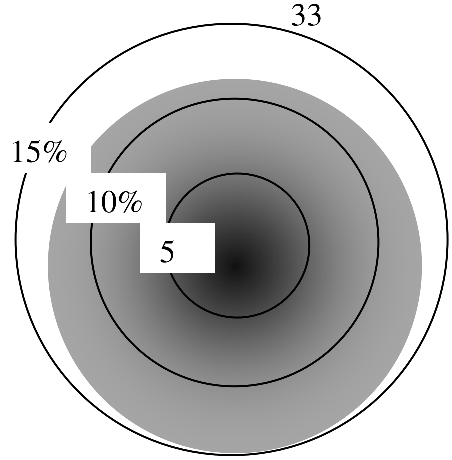



Figure 4 – A target diagram for the experiment described here.

**Conclusions:**

**1) Was the theory upheld? Support your answer by comparing the %difference to the %precision?** The experimental value of $\mu$=0.35 between copper and steel differed from the accepted value of $\mu_{acc}$=0.36 by about 3% (the percent difference). The precision of this experiment was about 13%. Since the percent difference was less than the precision, our experiment was in good agreement with the accepted value. This is represented by the target diagram to the right. The bullet is large because the experiment was not very precise. It is hanging a little bit low because the percent difference was a small negative number (small compared to the precision).

**2) What was the largest contributor to error? Defend your answer.** We expect that the largest contributor to error was *t*. When considering reaction times of perhaps 0.1s, this made typical errors for *t* of about 0.1/0.65 ≈ 15%. Furthermore, since our formula used $t^2$, we would expect this error to contribute twice as much. Averaging ten trials would tend to reduce the error by a factor $\sqrt{10} \approx 3$. It is reasonable to expect errors on the order of

15%×2 (because *t* was squared) ×1/3 (because ten trials were averaged) ≈ 10%

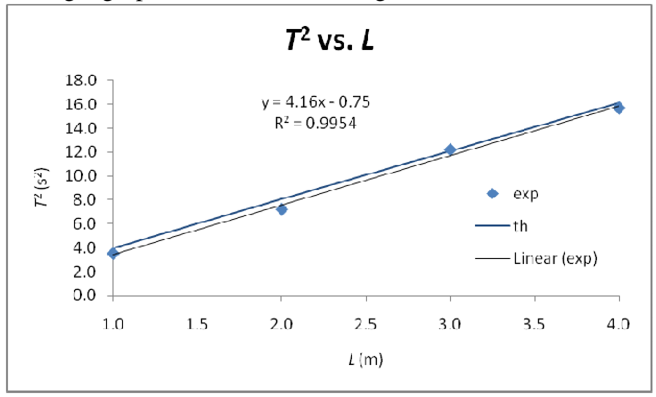
This is consistent with our estimate of the precision using the standard deviation.

**3) Suppose reaction time made you consistently decrease your measurement of *t*. Would you expect more positive or negative %differences?** A lower measurement of *t* would *increase* the second term in the equation for $\mu$ (see below). By subtracting a larger number we would see the experimental value of $\mu$ decrease. Since $\%diff = \frac{exp-th}{th} \times 100\%$, a smaller experimental value leads to a more negative *%diff*.

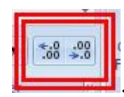
$$\mu = \frac{m_2 g - (m_1 + m_2)\frac{2h}{t^2}}{m_1 g}$$

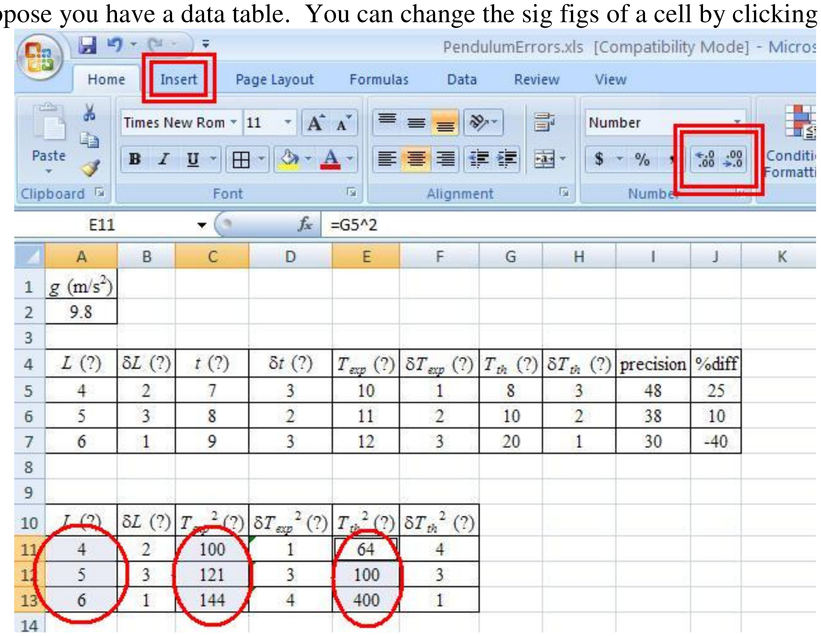
## Appendix J: Graphing

This appendix is designed to help you make a graph. After making a graph, check the following checklist:




- gridlines removed
- graph has title using variables (in italics) but no units in title
- axis labels have **variables in italics** with **units not italicized**
- graph fills the entire field (graph size should be about 1/3 to 1/2 a page)
- **no legend for single set of data** (a legend is only used if more than one thing is on a single graph)
- **if trendline is shown** there is an equation with $R^2$ value on the chart
- show the **experimental data as points only** (no connecting smooth line)
- **for graphs with both theoretical & experimental data**: theory is a smooth line (no points) while experiment is a only points (no line)
- the graph and data table are sized such that it fits on a single sheet of paper (not always possible)
- everything except the legend has appropriate subscripts and superscripts
- title is y-axis label versus x-axis label

Suppose you have a data table. You can change the sig figs of a cell by clicking these buttons .




| $g$ (m/s²) |
|---|
| 9.8 |

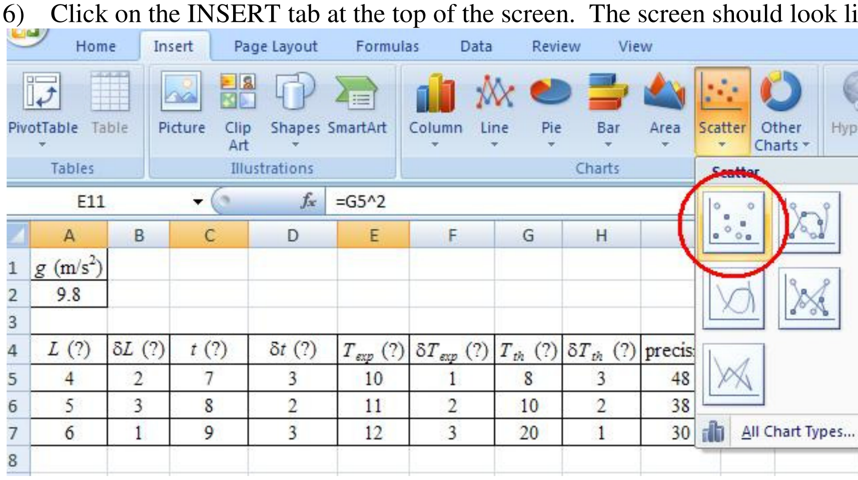
| $L$ (?) | δ$L$ (?) | $t$ (?) | δ$t$ (?) | $T_{exp}$ (?) | δ$T_{exp}$ (?) | $T_{th}$ (?) | δ$T_{th}$ (?) | precision | %diff |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 7 | 3 | 10 | 1 | 8 | 3 | 48 | 25 |
| 5 | 3 | 8 | 2 | 11 | 2 | 10 | 2 | 38 | 10 |
| 6 | 1 | 9 | 3 | 12 | 3 | 20 | 1 | 30 | -40 |

| $L$ (?) | δ$L$ (?) | $T_{exp}^2$ (?) | δ$T_{exp}^2$ (?) | $T_{th}^2$ (?) | δ$T_{th}^2$ (?) |
|---|---|---|---|---|---|
| 4 | 2 | 100 | 1 | 64 | 4 |
| 5 | 3 | 121 | 3 | 100 | 3 |
| 6 | 1 | 144 | 4 | 400 | 1 |

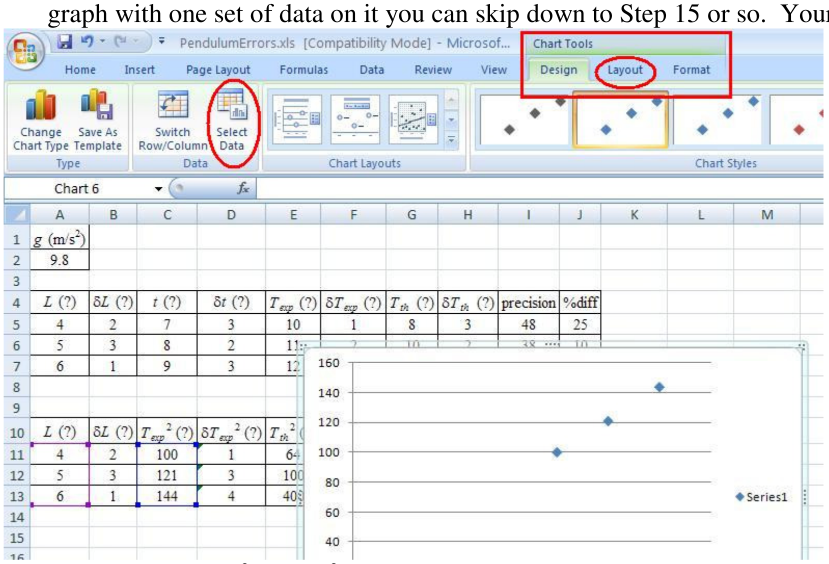
Notice the following things in the data table.

- variables are italicized
- units are not italicized
- the δ symbol and parentheses are not italicized
- the %precision and %difference do not have units (everything else does have units)!
- the cells which will contain data are centered with borders
- $g$ is a constant (the same for all experiments) so it is written at the top in a separate table
- subscripts and superscripts are appropriately used

1) Now make a graph of $T_{exp}^2$ vs $L$ and $T_{th}^2$ vs $L$ on the same graph. To do this you must first highlight the columns of data (numbers only) you want to graph. If they are all right next to each other you can highlight all three columns in a single swoop. If the three columns are all adjacent you will need to do steps 6 & 7 then skip to step 14.
2) If the data columns are all over the place (not right next to each other) you should use the CTRL button on the bottom left of the keyboard. Hold down the CTRL button.
3) Depress the left mouse button and drag it over the numbers of the first column. DO NOT INDIVIDUALLY SELECT THE NUMBERS (it screws up the graph).
4) WHILE THE CTRL BUTTON IS STILL HELD DOWN, highlight the next column ($T_{exp}^2$) of data in a similar way (dragging mouse over the numbers with left mouse button depressed).
5) Now you can let go of the CTRL button.
6) Click on the INSERT tab at the top of the screen. The screen should look like the one below.

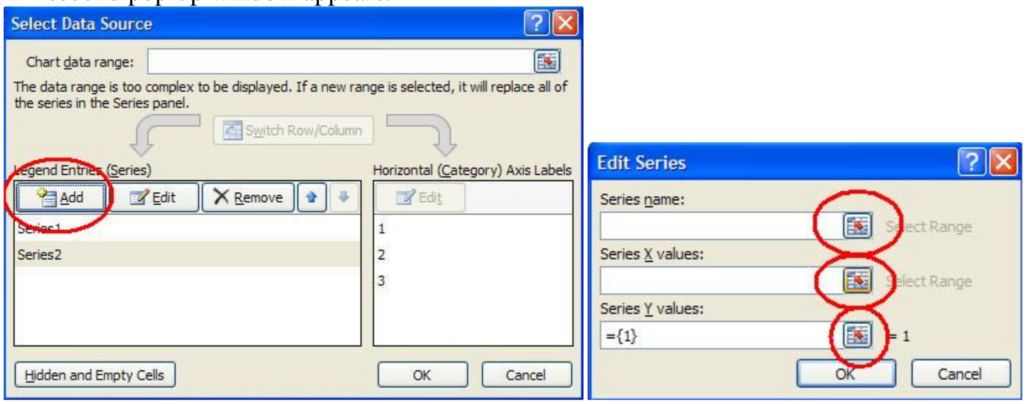



7) Now click on the SCATTER button to make a plot. Select the option that shows only dots. If you only want a graph with one set of data on it you can skip down to Step 15 or so. Your screen should look like this:

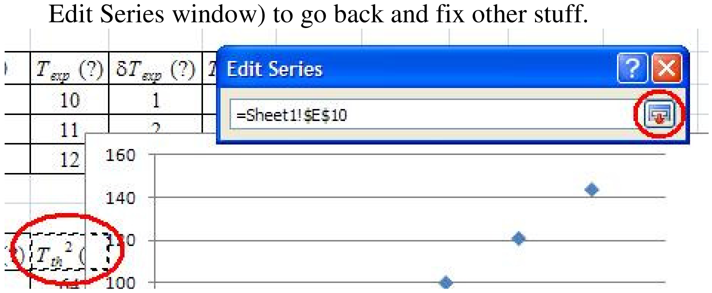



8) To make both $T_{exp}^2$ and $T_{th}^2$ appear on the same graph you must first “add a series” by clicking on the SELECT DATA button at the top left of the screen. Other important buttons are seen at the top right under CHART TOOLS.

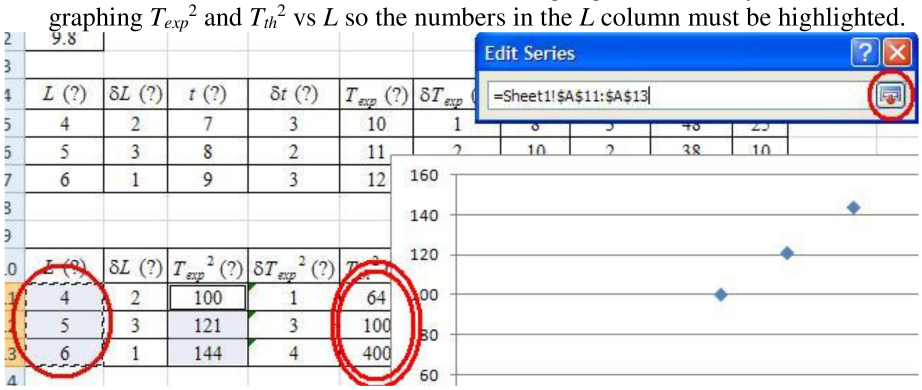
9) When you click the SELECT DATA button, a pop-up windows appears. Click on the button to ADD and the second pop-up window appears.




10) The three buttons on the right (in the second pop-up) allow you to input information from the table using the mouse. Click the top button for Series Name and you'll see the screen change to the one below. The name of the series is simply the column heading. Click on the column heading that you want to add. In this case we are adding the column of data for $T_{th}^2$ so click on that cell. Then click on the button shown (circled at far right in Edit Series window) to go back and fix other stuff.



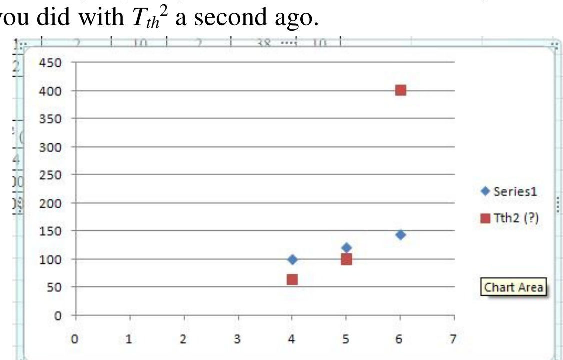

11) Now click on the "Series X values" button. Highlight the values you want on the x-axis. In this case we are graphing $T_{exp}^2$ and $T_{th}^2$ vs $L$ so the numbers in the $L$ column must be highlighted.



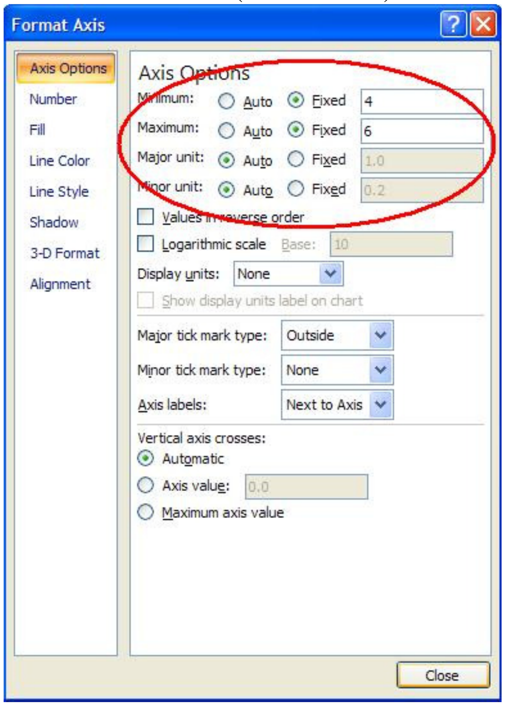

12) Click the go back button in the Edit Series window. Hit the Series Y value button and highlight the appropriate y-values. In this case the y-values we need are $T_{th}^2$. Click to go back and hit ok.

13) Try to fix up the remaining series title by first highlighting SERIES 1 and then clicking edit. You should be able to then select the $T_{exp}^2$ cell just like you did with $T_{th}^2$ a second ago.

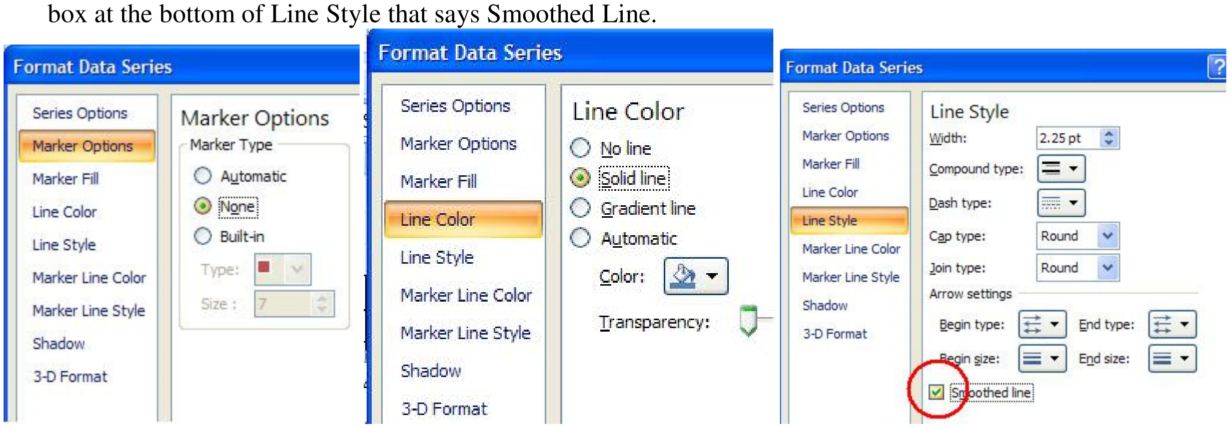
14) Your graph should look like this:



15) Do get rid of those annoying horizontal gridlines simply click on one of the lines and hit delete.
16) Notice how there is a lot of wasted space. Fix this by right clicking on the bottom axis. In the pop-up select FORMAT AXIS (at the bottom). You should see this screen:

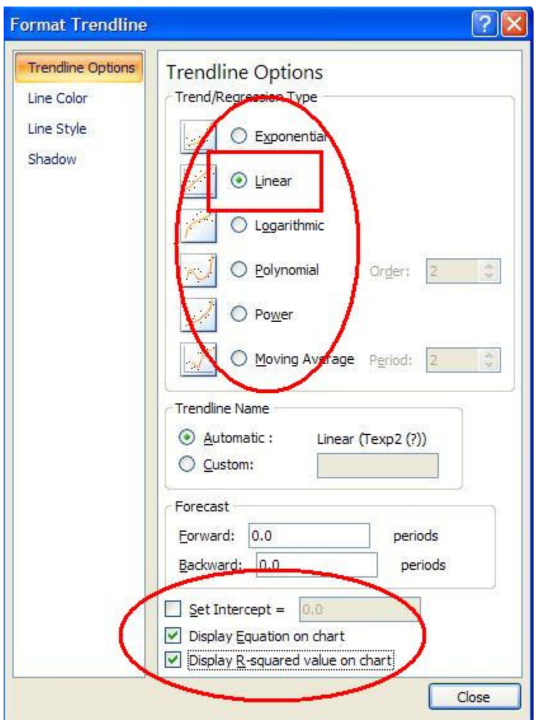



17) To make the graph take up less space click on the two buttons that says Fixed (one for maximum and one for minimum). You can then enter in your smallest and largest values for the x-axis manually. In this case the smallest value of $L$ is 4 and the largest is 6. I typed these in so the graph would be as big as possible. Click close when you are done. You could similarly modify the y-axis if it needs it but in this case it looks fine.
18) Now we need to make it easy to quickly distinguish the theory from the experiment on the graph. To do this we can make the theory a smooth line with no points and leave the experiment as points with no line.
19) To do this you must right click on one of the theoretical data points (in this case the red ones). In the pop-up click on Format Data Series and another pop-up appears. There are three tabs you need to visit: Marker Options, Line Color, and Line Style. Click NONE at the Marker Type, Solid Line at Line Color, and check the box at the bottom of Line Style that says Smoothed Line.

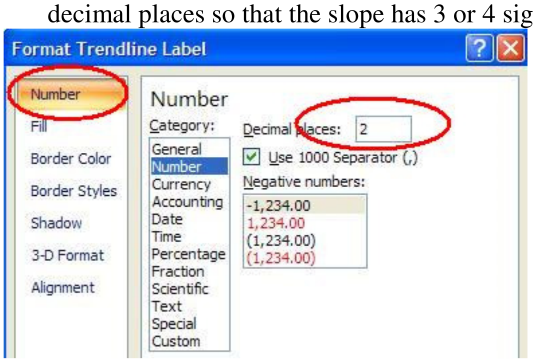



20) Almost there…but what are we graphing…and what is each axis? To put on a title and axis labels you must click on the Layout tab at the very top of the screen (see the picture at step 2).
21) To add a title click Chart Title and select the option for Above Chart. Enter the title using variables and a description if needed. $T^2$ vs $L$ should be fine for this. To make the $L$ and the $T$ italicized you can highlight them and hit CTRL+"I".

22) Now click on the Axis Titles button and add both the Primary Horizontal Axis Title (use Title Below Axis) and Primary Vertical Axis Title (use the Rotated Title).
23) Remove the bold in each title by highlighting the field and hitting CTRL+“B”.
24) Add in the appropriate text. Get the variables in italics using CTRL+“I”.
25) Get the superscripts and superscripts by first highlighting that part to super or subscript. Then right-click the Format Axis Title in the pop-up. Lastly click on superscript or subscript as appropriate.
26) Changing the format of your legend (the box on the right) can be done somehow but I can’t remember how to do it right now. If you figure it out be sure to write down how it is done for me so I can type it up. I’ll give you some extra credit (not much) on a lab score if you show me how.
27) To add a trendline, first right click on an experimental data point (in this case a blue point). When the pop-up appears you must first select the right type of line.
28) Is it exponential, linear, quadratic (polynomial of order 2), etc? In this case we know that $T = 2\pi[L/g]^{1/2}$ which implies that $L$ is proportional to $T^2$. This implies that a plot of $T^2$ vs $L$ should be linear! A graph of $T$ vs $L$ would require a square root fit (which is not available in Excel). Select the linear option for this example.
29) Also click two buttons at the bottom of the pop-up that say “Display Equation on Chart” and “Display R-squared Value on Chart”. An $R^2$-value close to one usually implies that the data correlates well to the type of mathematical function.




30) To change the sig figs of your trendline equation, right click on the trendline. CLICK ON THE TRENDLINE SOMEWHERE BETWEEN DATA POINTS. You might have to try this a few times as it is hard to get the mouse exactly over the trendline. Click on the Format Trendline option. Use the Number tab to get the right number of decimal places. For this example let’s use 2 decimal places. Usually you’ll want to have enough decimal places so that the slope has 3 or 4 sig figs. Now check your graph with the initial checklist.



## Appendix K: Quickest Estimate of Error

The simplest method for estimating error is to add the percent errors of all measurements together. While this is in no means a robust figure that you should bet your life on, it often is very close to a more rigorous estimate of errors. It is used to get a ballpark figure for the error in an experiment.

Suppose the you are told that

$$F = \frac{GmM}{r^2}$$

Furthermore you collect the data as shown in the table below (last cell calculated from first four):

| $M$ (kg) | $m$ (kg) | $G$ (N·m$^2$/kg$^2$) | $r$ (m) | $F$ (nN) |
|---|---|---|---|---|
| 5.0 | 5.0 | 6.67E-11 | 0.20 | 42 |

Notice that it made sense to use the nano prefix for the force. It would then be reasonable to assume the following errors for each of your measurements:

| $\delta M$ (kg) | $\delta m$ (kg) | $\delta G$ (N·m$^2$/kg$^2$) | $\delta r$ (m) | $\delta F$ (nN) |
|---|---|---|---|---|
| 0.1 | 0.1 | NA | 0.01 | ???? |

Note: there is no assumed error for $G$ because it is a well-studied constant from your physics text. For the others I simply figured that the error is in the rightmost decimal place. Notice that these errors still have units while percent errors do not have units. The percent errors are:

| %err $M$ | %err $m$ | %err $G$ | %err $r$ | %err $F$ |
|---|---|---|---|---|
| $\frac{0.1\text{kg}}{5.0\text{kg}} = 2\%$ | $\frac{0.1\text{kg}}{5.0\text{kg}} = 2\%$ | 0% | $\frac{0.01\text{m}}{0.20\text{m}} = 4\%$ | 2%+2%+0%+4%+4%=12% |

Notice that %'s have no units! Notice that to get the %error in $F$ I simply added the %errors of all of the measurements that were used to calculate $F$. The %error in $r$ contributes twice as much as the others because it is squared! If a term was under a square root then its %error would only contribute ½ as much to the total %error. This gets the right order of magnitude for the error (about 10%). I can now see that

| $\delta F$ (nN) |
|---|
| $12\%$ of $42 = 5.04 \approx 5$ |

One could then choose to write

$$F = 42 \pm 5 \text{ nN}$$

**Appendix L: An Example of Error Analysis for a Typical Experiment:**

An equation in physics says that $a = \frac{2x}{t^2}$. You measure the position $x$ and the time $t$ five times each. You make the following data table:

Table 1: Position versus time information when a steel ball was dropped from various heights.

| $x$ (m) | $t$ (s) | $a$ (m/s$^2$) |
|---|---|---|
| 4.95 | 1.01 | 9.7 |
| 4.95 | 1.03 | 9.3 |
| 21.10 | 2.12 | 9.4 |
| 21.10 | 2.11 | 9.5 |
| 40.87 | 2.93 | 9.5 |
| 40.87 | 2.97 | 9.3 |
| | **avg** | 9.4 |
| | **stdev** | 0.2 |

In this case the value of $a$ determined by the experiment is $a = 9.4 \pm 0.2\,\frac{\mathrm{m}}{\mathrm{s}^2}$. Notice that the number of significant figures in the average is determined by the standard deviation. The standard deviation is rounded to one significant figure which ends up being the tenths column. This causes all the calculated values of $a$ to be rounded to the tenths column.

So how does this value compare to the accepted value of $g$ given by 9.8 m/s$^2$? At first glance you might think the experiment is a failure and that the experiment is not in good agreement with the theory. Let's determine this more precisely with some numerical work…
The %difference in this experiment is given by:

$$\%difference = \frac{9.4 - 9.8}{9.8} = -4\%$$

The %precision is given by:

$$\%precision = \frac{0.2}{9.4} = 2\%$$

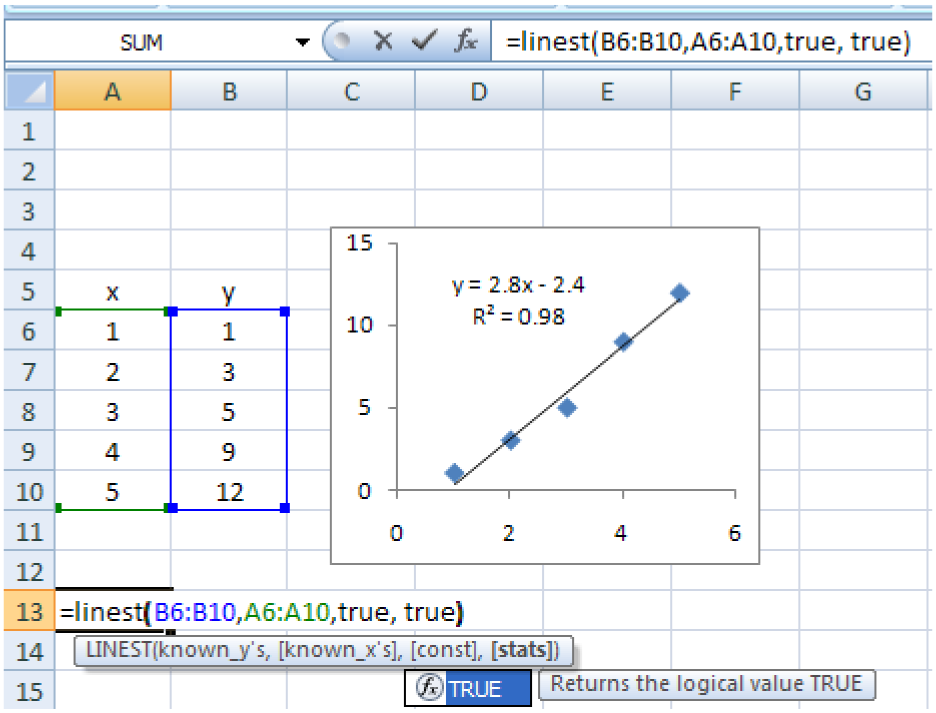
Since the %difference is larger than the %precision one could argue that the experiment does not agree with the accepted value.

Also note that the precision is less than 5% which indicates the equipment used was quite good for the task at hand. This indicates that something (perhaps friction) was not considered in the theory used in the experiment. One would think that friction would tend to slow an object and increase the times in the experiment. This would tend to decrease the values found for $a$ in the experiment (because $t$ is in the denominator). This explains the negative percent difference.

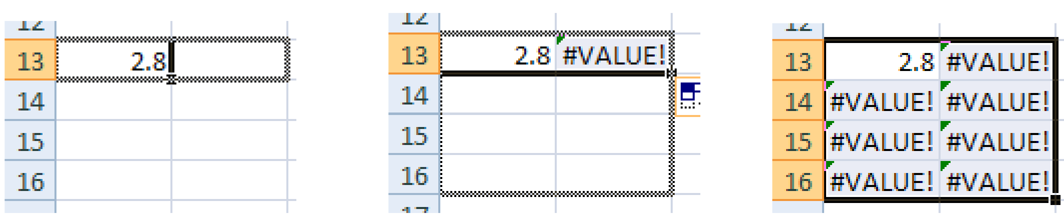
As a last comment, consider that 68% of the time a measurement is within $1\sigma$ of the mean, 90% of the time the value is within $2\sigma$ of the mean, and 99% of the time the value is within $3\sigma$ of the mean. This relates to confidence limits.

## Appendix M: Find the Error in the Slope of a Graph Using LINEST

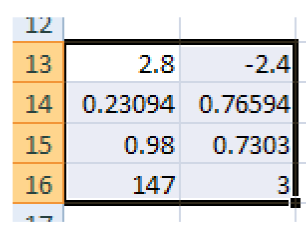
First get a set of data showing both an *x* and a *y* column. In a cell somewhere below that column of data type "=linest(…)" as seen in the figure below. Notice that you highlight the *y*'s first then, separated by a comma, the *x*'s. The two words "true" are used to tell excel to NOT to set the slope to zero and to calculate the regression statistics.




Mouse over the bottom, right-hand corner of cell A13 until cursor changes to the "fill down cursor". Drag to the right one cell using the fill down cursor (left-most figure below). Then drag down four cells making the 8 cell block seen below (right two figures below).

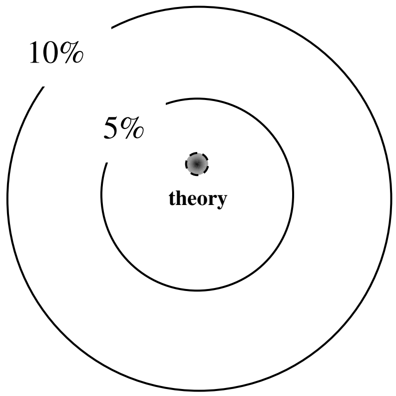



While cells are still highlighted press the F2 button. Then press CTRL-SHIFT-ENTER and notice that the cells have actual numbers in them. Notice that the top two numbers are the slope (2.8) and intercept (-2.4) respectively. The number directly below the slope is the error in the slope (0.23094) while the number directly below the intercept is the error in the intercept (0.76594).



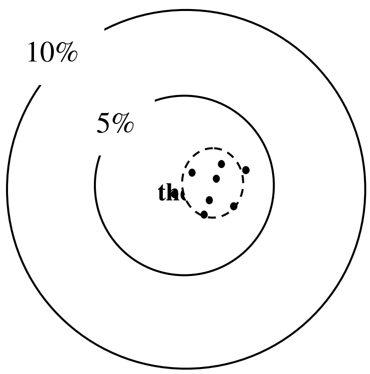
## Appendix N: Target Diagrams

**Target Diagrams for different types of experiments**



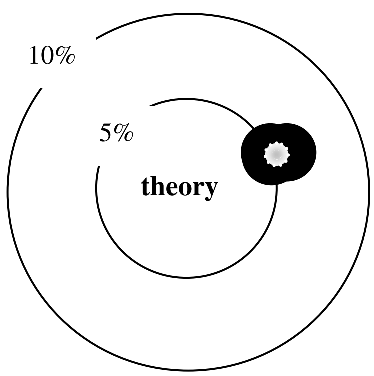

The figure at left shows a “target diagram” for an experiment with a **single measurement**. Notice that:

1) The theoretical value is the bull’s-eye of the target
2) The percent difference is the distance of the bull’s-eye from the target.
3) The %precision (determined from propagation of reading errors) is the radius of the bullet.
4) Since the %difference (distance from bull’s-eye) is larger than the %precision (size of the bullet) the experimental results are inconsistent with the theory (the bullet hole doesn’t touch the bull’s-eye).




The figure at left shows a target diagram for an experiment with **multiple measurements with a LARGE standard deviation and small reading error**. Notice that:

1) The theoretical value is the bull’s-eye of the target.
2) The several measurements look like a shotgun blast.
3) The individual measurements are very good (perhaps 4 or 5 sig figs) but the standard deviation ($\sigma$) indicates only 2 or 3 sig figs.
4) The distance from the bull’s-eye to the center of the shotgun blast is determined by the percent difference.
5) The %precision (determined by $\sigma$) is the radius of the shotgun blast.
6) A few measurements actually lie outside the standard deviation.
7) Since the %difference (distance from bull’s-eye) is smaller than the %precision (size of the bullet group) the experimental results are consistent with the theory (the shotgun blast touches the bull’s-eye).




The figure at left shows another possible target diagram for an experiment with **multiple measurements with a SMALL standard deviation and large reading error**. Notice that:

1) The individual measurements are very bad (perhaps 2 or 3 sig figs) but the standard deviation ($\sigma$) indicates 4or 5 sig figs. It is hard to see the multiple measurements because they are almost on top of each other.
2) The distance from the bull’s-eye to the center of the shotgun blast is determined by the percent difference.
3) The %precision is determined by propagation of the reading errors and NOT $\sigma$.
4) Since the %difference (distance from bull’s-eye) is larger than the %precision (size of the bullet group) the experimental results are not consistent with the theory (the bullets don't touch the bull’s-eye).

$x$= measured value ± measuring error = $x$±$\delta x$ Fractional error = $\frac{\delta x}{|x|}$ Percentage error=$\frac{\delta x}{|x|} \times 100\%$

%difference = $\frac{experimental-theoretical}{theoretical} \times 100\%$ and it can be negative %precision= $\frac{\delta exp}{exp} \times 100\%$

**Quickest Error Estimate:**

1) First measure $a$ and $t$ and use them to calculate $x$ using $x=\frac{1}{2}at^2$.
2) Then determine $\delta a$ and $\delta t$ using the rightmost column of the measured data and keep only one sig fig.
3) Next determine the %errors for $a$ and $t$ using the formulas at the top of this page & keep only one sig fig.
4) Combine these errors to get the percent error in $x$ (be sure to double count any squared terms and half count any square-rooted terms).
5) Use the %error in $x$ to get $\delta x$ AND round it to one sig fig.
6) Lastly, make sure to match the column of $x$ to the column of $\delta x$.

| $a$ (m/s$^2$) | $t$ (s) | $x$ (m) |
|---|---|---|
| 4.00 | 5.0 | 50 |
| $\delta a$ (m/s$^2$) | $\delta t$ (s) | $\delta x$ (m) |
| 0.01 | 0.1 | 2 |
| %err $a$ | %err $t$ | %err $x$ |
| 0.3 | 2 | 4 |

**Error propagation formulas:**

When adding OR subtracting errors assume errors add in quadrature.

If $q=x+y-z$ then $\delta q=\sqrt{\delta x^2+\delta y^2+\delta z^2}$.

If $q=Bx$, where $B$ is known exactly (for instance $B$ is a well-tested physics constant):

$$\delta q = B\delta x$$

If $q$ is a product/quotient given by $q=\frac{xy}{uz}$ use the following formula:

$$\frac{\delta q}{q}=\sqrt{\left(\frac{\delta x}{x}\right)^2+\left(\frac{\delta y}{y}\right)^2+\left(\frac{\delta u}{u}\right)^2+\left(\frac{\delta z}{z}\right)^2}.$$

If $q$ is a single number to a power given by $q=x^n$ use the following formula:

$$\frac{\delta q}{q}=n\frac{\delta x}{x}.$$

If $q$ is a product/quotient with powers (say $q=\frac{x^{1/2}y^{-9}}{u^{-5/3}z^4}$) the above formula is modified to:

$$\frac{\delta q}{q}=\sqrt{\left(\frac{1}{2}\frac{\delta x}{x}\right)^2+\left(9\frac{\delta y}{y}\right)^2+\left(\frac{5}{3}\frac{\delta u}{u}\right)^2+\left(4\frac{\delta z}{z}\right)^2}.$$

**Standard Deviation (for several measurements which should, in theory, produce identical results):**

The average and standard deviation of a finite set of measurements (in this example $N$=5) is best given by

$$\bar{x} = average\ of\ N\ values\ of\ x=\frac{\sum_{i=1}^{N}x_i}{N}=\frac{x_1+x_2+x_3+x_4+x_5}{5}$$

$$\sigma_x = standard\ deviation of\ x=\sqrt{\frac{\sum\left(x_i-x_{avg}\right)^2}{N-1}}$$

$$=\sqrt{\frac{\left(x_1-x_{avg}\right)^2+\left(x_2-x_{avg}\right)^2+\left(x_3-x_{avg}\right)^2+\left(x_4-x_{avg}\right)^2+\left(x_5-x_{avg}\right)^2}{4}}$$

The error in the average of the values is given by one of the following methods:

1) $\bar{x}$=average of $N$ values of $x$ ($\bar{x}=\frac{\sum_{i=1}^{N}x_i}{N}$) then $\delta\bar{x}=\frac{\delta x}{\sqrt{N}}$.
2) $\delta\bar{x} = the\ standard\ error = \sigma_x$. Expect a new measurement lies within $\sigma_x$ of $\bar{x}$ about 68% of the time.

**If $\sigma_x$ is extraordinarily small,** use method 1 to compute the error in the average.